in Fanout-Free
ircuits

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t, in a simple manner, a test
combinational circuit with *n*
gnoses) nonequivalent single
over the upper bound in [1,
r of primary input gates) and
of 2*n* for the least number of
nequivalent single faults.

ng single faults, fanout-free
, test set.

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talking about the standard
uld consult [1], [2], or [3] if

ingle faults if given any two
means that they differ on
n which the two faults differ.
etter than to determine the
orithm we present produces
$n + 1$ where n is the number
ional circuit. This is an im-
[1], [3], and [5].
s is known for fanout-free
01]), we will not bother to
cular faults. Such a relation-
e and can be worked out by

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it, we use $T^u(T^z)$ to denote
cuit has the value 1(0). The
assume that we know what
ok like on each of the input
a single fault diagnostic test
tput gate. Furthermore, to
out the algorithm in detail
gate G.
input lines of G, and $T_1, \cdots,$
test sets for the circuits for
n as output lines. Further-
ote the Boolean function
r which l_i is the output line
l_i. Note that each $f_i \not\equiv 0, 1$;
ce of faults, the output of G should produce the function $f = f_1 \wedge \cdots \wedge f_k \not\equiv 0, 1$. One additional notational convention is that for $t_i \in T_i$ we use $(t_1, \cdots, t_k)$ to denote the test that consists of simultaneously applying t_i to the inputs feeding l_i for all i.

With everything as described above, let u_i be an arbitrary member of T_i^u and define

$$T^u = \{(t, u_2, \cdots, u_k) \mid t \in T_1^u\} \cup \cdots$$
$$\cup \{(u_1, \cdots, u_{k-1}, t) \mid t \in T_k^u\}$$
$$T^z = \{(t, u_2, \cdots, u_k) \mid t \in T_1^z\} \cup \cdots$$
$$\cup \{(u_1, \cdots, u_{k-1}, t) \mid t \in T_k^z\}$$

and

$$T = T^u \cup T^z.$$

Note that T is very close to the set produced by Procedure 2 in [5].

We now have the following theorem.

Theorem: T is a single fault diagnosing test set for the circuit ending at G. $|T^u| = \sum_{i=1}^k |T_i^u| - k + 1$ and $|T^z| = \sum_{i=1}^k |T_i^z|$. Thus, $|T| = \sum_{i=1}^k |T_i| - k + 1$.

Proof: If a single fault or no fault occurs in the circuit, then the output of G is one of the functions 0, 1,

$$g_{11} \wedge f_2 \wedge \cdots \wedge f_k,$$
$$g_{12} \wedge f_2 \cdots \wedge f_k, \cdots, g_{1m_1} \wedge f_2 \wedge \cdots \wedge f_k,$$
$$f_1 \wedge g_{21} \wedge f_3 \wedge \cdots \wedge f_k, \cdots, f_1 \wedge g_{2m_2} \wedge f_3 \cdots \wedge f_k, \cdots,$$
$$f_1 \wedge f_2 \wedge \cdots \wedge f_{k-1} \wedge g_{km_k}, \quad f_1 \wedge \cdots \wedge f_k$$

where g_{ip} $(p = 1, \cdots, m_i)$ are the nonequivalent nonzero functions which can occur as a result of a single fault in the circuit having l_i as an output line (see [2], [4]).

Since each $f_1 \wedge \cdots \wedge g_{ip} \wedge \cdots \wedge f_k$ is 0 on any element of the form $(u_1, \cdots, u_{r-1}, t, u_{r+1}, \cdots, u_k)$ with $r \neq i$ and $t \in T_r^z$ and 1 on some element of the form $(u_1, \cdots, u_{i-1}, t, u_{i+1}, \cdots, u_k)$ with $t \in T_i$ (since T_i distinguishes g_{ij} from 0), T clearly distinguishes between 0, 1 and the other faults. Since $T^z \neq \emptyset \neq T^u$, T diagnoses the faults 0, 1.

We now show that if θ, γ are two distinct functions in the list given at the start of the proof such that $\theta, \gamma \not\equiv 0, 1$, then some element of T causes them to assume different values. There are two cases to consider.

Case 1: θ and γ differ only in the ith conjunct. Here it is clear that T distinguishes between θ and γ since some $t \in T_i$ distinguishes between the ith conjunct of θ and γ, whence $(u_1, \cdots, u_{i-1}, t, \cdots, u_k)$ distinguishes between θ and γ. In particular, this shows that T detects all single faults (it actually detects all multiple faults; see [2], [4]).

Case 2: θ and γ differ in the ith and jth conjuncts with $i \neq j$. Thus, we may assume that $\theta = f_1 \wedge \cdots \wedge g_{ip} \wedge f_{i+1} \wedge \cdots \wedge f_k$ and $\gamma = f_1 \wedge \cdots \wedge g_{jq} \wedge f_{j+1} \wedge \cdots \wedge f_k$. Note that θ is 0 on

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puter Sciences, IBM T. J. Watson

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$\cdots, u_k) \mid t \in T_i^z\}$

$\cdots, u_k) \mid t \in T_i^z\}.$

evious sentence is all of T^z,
between θ and γ or θ and
e done. In the second, we

ally 0 on T^z, then $g_{ip} \equiv 0$
f_i, $g_{ip} \equiv 0$ on T_i^z and T_i
t l_i, there exist t_1, $t_2 \in T_i^u$
Let $t_1^* = (u_1, \cdots, u_{i-1}, t_1,$
$u_{i+1}, \cdots, u_k)$. If $g_{jq}(u_j) = 0$,
$) = 1$, then $\theta(t_2^*) = 0$ while
shes between θ and γ.
m of the $|T_i^z|$ since all the
int, while $|T^u|$ is just the
the element $(u_1, \cdots, u_k)$
to create T^u. ☐
nal circuit with n primary
est set containing exactly

uction on n. For $n = 1$ or 2,
the case of the AND gate
ose T_i so that $|T_i| = n_i + 1$
ts feeding the line l_i. By the
$1 = (\sum_{i=1}^{k} n_i) + 1 = n + 1.$
ssentially identical. ☐

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