and we immediately obtain conditions on the coefficients for the quartic to be the square of a quadratic.

Finally some generalisations, suppose $P(x)$ is a polynomial of degree $d = pq$ where p, q are integers $\neq 1$. We can investigate the possibility that $P(x)$ is the pth power of a polynomial of degree q or, alternatively, that it is the qth power of a polynomial of degree p. For example a certain sextic might be the cube of a quadratic or the square of a cubic. In the first case

$$P(x) = Q(x)^3,$$

where

$$Q = \frac{P^{(iv)}}{72} - \left(\frac{P^{(v)}}{360}\right)^2.$$

In the second case

$$P(x) = R(x)^2,$$

where

$$R = \frac{1}{12}\left\{P''' - \frac{P^{(iv)}P^{(v)}}{960} + \frac{3}{4}\left(\frac{P^{(v)}}{120}\right)^3\right\}.$$

Clearly once we get beyond the sextic the formulae are going to get more complicated. The formula for expressing a polynomial of degree $2p$ as the pth power of a quadratic is relatively easy to find—I leave it as an exercise for the reader—but I baulk at trying for the square root of the polynomial. Perhaps readers will have other ideas and experiences of how to overcome software inadequacies and/or hardware failures.

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75.3 Making a Golden Rectangle by paper folding

The Golden Ratio $((1 + \sqrt{5})/2 \approx 1{\cdot}618)$ is a number with many interesting mathematical properties. The purpose of this short note is to show how you can construct a Golden Rectangle using just a sheet of 8·5″ × 11″ paper without any other tools. A Golden Rectangle is a rectangle such that the ratio of one dimension over the other is the Golden Ratio.

One problem in creating a Golden Rectangle out of an 8·5″ × 11″ sheet of paper is that the ratio of length/width is only about 1·29. Step 1 of the construction makes the paper narrow enough so we can create a Golden Rectangle from it. If for your sheet of paper the ratio of the longest dimension to the shortest dimension is greater than the Golden Ratio, then you can begin with Step 2.

1. Place the paper before you so its long dimension is horizontal. Make a horizontal fold in the paper in the exact middle of the sheet by folding the sheet in half. Fold the paper along the crease several times until the paper will tear readily along the crease. The final result will be a long thin rectangle *ABCD*.
2. Fold corner *A* so it rests on *DC* as shown in Fig. 1. Let *E* be the new point created on *AB* where the paper is folded.
3. Using *E* as a guide, create a vertical crease at point *E*. Let *F* be the point where the crease touches *DC*. In Fig. 2 the dashed line *EF* represents the crease, and the figure *AEFD* is a perfect square.
4. Now fold the paper so that *AD* perfectly coincides with *EF*. This creates a vertical crease in the middle of the square. Mark the ends of this crease with *G* and *H* as shown in Fig. 3.
5. Fold *HC* so that it runs through *E* and mark *HC* with a little fold where it intersects *E* as shown in Fig. 4. Call this point *J*.
6. Make a vertical crease at *J* by folding the paper. Call the other end of the crease *I*. Fig. 5 shows all the vertical creases created so far.
7. Fold the paper along *IJ* sufficiently many times to ensure that it rips smoothly along *IJ*, then rip it along *IJ*. The final rectangle, *AIJD* in Fig. 6, is a Golden Rectangle.

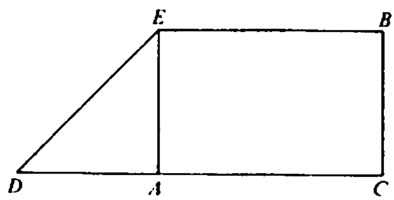

FIGURE 1. The first step in making a square.

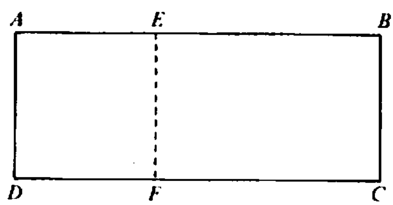

FIGURE 2. Creating a perfect square.

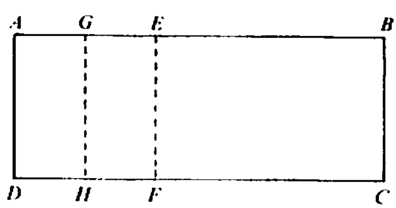

FIGURE 3. Dividing the square in half.

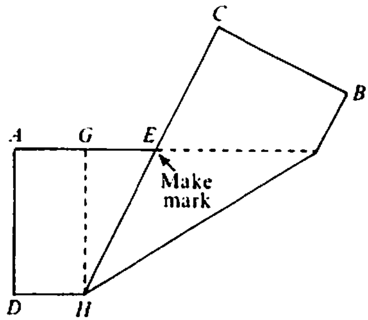

FIGURE 4. Marking the edge DC.

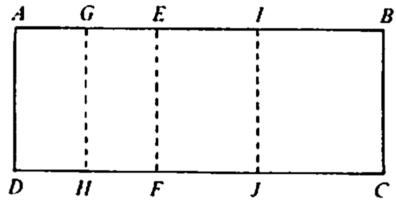

FIGURE 5. All the vertical creases in $ABCD$.

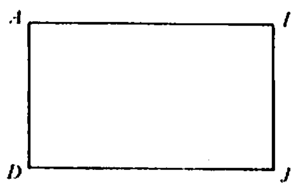

FIGURE 6. Finally, a Golden Rectangle.

To see that $AIJD$ of Fig. 6 is a Golden Rectangle, study Fig. 4 and use x to denote the distance DH. For an 8·5″ × 11″ piece of paper it is easy to see that $x = 2{\cdot}125''$. In terms of x, GE has length x and GH has length $2x$. Since HGE is a right triangle, the Pythagorean Theorem shows that $HE = HJ = x\sqrt{5}$. Thus, the size of DJ is $x(1 + \sqrt{5})$ and since the size of AD is $2x$ the ratio DJ/AD is the Golden Ratio.

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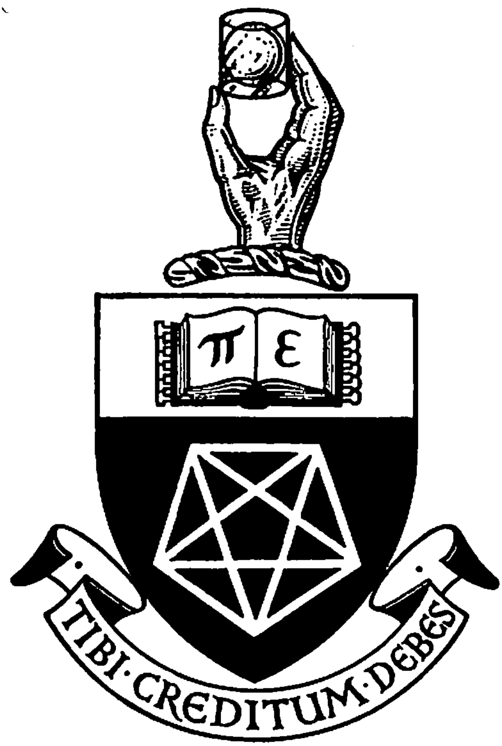

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