NUMERICAL TIC–TAC–TOE – II

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ABSTRACT

Part I of this article discussed two very interesting variations on Tic-Tac-Toe that use numbers instead of X's and O's [1]. These variations were discovered by Ron Graham and P. H. Nygaard. Part I presents analyses of both games along with several open problems and the history of these games. Part II presents a winning strategy for Odd in Graham's game.

1. Ron Graham's Game

Ron Graham's Game uses the standard 3 X 3 Tic-Tac-Toe board. Instead of calling the players X and O, we call them Odd and Even. Odd gets the numbers 1, 3, 5, 7, and 9, while Even gets the numbers 2, 4, 6, and 8. Odd goes first, after which the players take turns placing one of their numbers in the empty cells of the standard Tic-Tac-Toe board. Numbers may be used only once. The object of the game is to be the player who completes a line that sums to 15. As in Tic-Tac-Toe, a line is a row, column, or diagonal.

Completing a line means being the player who puts the final number in it so the entire line sums to 15. Players are allowed to use numbers placed by the opponent to reach the sum of 15. Note that once a line contains two numbers whose sum is 15 or greater, there is no way to complete that line, although filling in the remaining cell might be necessary to complete a different line.

The setup for playing Graham's Game is quite simple. Start out with a piece of paper which has the familiar Tic-Tac-Toe board on it. To one side write a row with the numbers 1 3 5 7 9 in it and below it write a row with the numbers 2 4 6 8 in it. As the players use the numbers they must cross them off. Figure 1 shows the setup for playing Graham's Game.
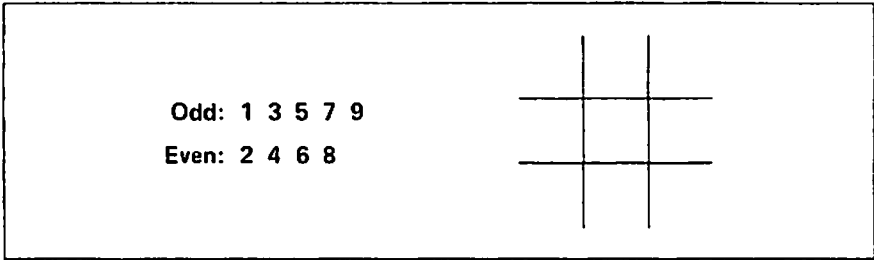
192

Odd: 1 3 5 7 9

Even: 2 4 6 8

Figure 1. The setup for playing Graham's Game.

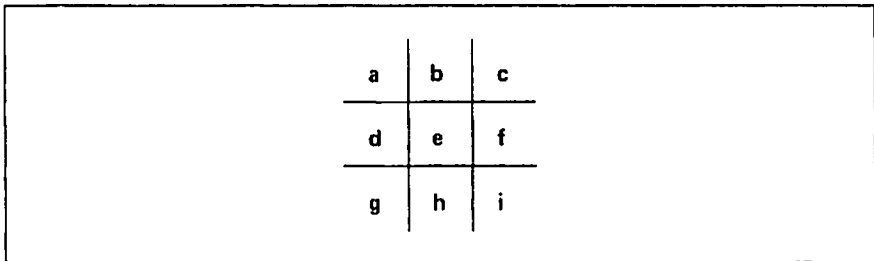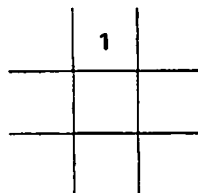| a	b	c
d	e	f
g	h	i

Figure 2. Labeling the positions on a Tic-Tac-Toe Board.

2. A Winning Strategy for Graham's Game

To explain the following strategy we must first introduce a compact system of labeling the Tic-Tac-Toe board. One possible labeling is shown in Figure 2. The notation in Figure 2 is similar to chess notation and was suggested to me by Danny Kopec. It is clearer than the numerical notation I was originally using since it distinguishes between values on the board and positions.

Odd can win by putting 1 into position b. By the reduction discussed in the previous section, putting 1 into positions d, f, and h should also lead to wins. Again, the reductions discussed in Part I of this article show that using 9 = 10 – 1 should produce the same results. Thus, after the first move, the board should look like:

1

2a9h	2c9h	2d5c	2e3a	2f5a	2g3a	2h9a	2i3h
4a9h	4c9h	4d5c	4e7a	4f5a	4g7a	4h9a	4i7h
6a7c	6c7a	6d9a	6e9h	6f5a	6g9f	6h7e	6i9h
8a3c	8c3a	8d9a	8e9h	8f3a	8g9f	8h3e	8i9h

Figure 3. Odd's second move.

Odd's Second Move

The second move for Odd is given in Figure 3, where each sequence of four characters reads as

Even _ Number Position Odd _ Number Position

For example, the first entry means that if Even puts the 2 in position a, then Odd should put the 9 in position h. Note that all thirty-two possibilities (four responses in eight positions) are present in the table. Also note that for each move Even makes, Odd needs to have only one response.

Odd's Third Move

After Odd's second move, there will be three numbers on the board. Furthermore, if Odd follows the strategy given in Figure 3, only one of thirty-two games will be possible at this time. It is impossible for Odd to win after only two moves since two odd numbers and one even number cannot add up to 15.

In each case, Even can respond in 3 X 6 = 18 ways. As before, for each move that Even makes, Odd needs to have only one response. The correct responses are listed in Figures 4(a) and 4(b). To use this table, first look up the current configuration in the leftmost column. Each triple of digits gives one of the rows, thus the sequence 010 000 032 corresponds to the configuration

```
    |  1 |
----+----+----
    |    |
----+----+----
    |  3 |  2
```

where zeros indicate blank cells. Odd's responses are coded as before, except that this time an extra character (W or C) is added to the end of the 4-tuple indicating whether Odd Wins on that move or whether the game is to Continue.

Odd's Fourth Move

There are 181 configurations that must be analyzed after Odd's third move. These result because Odd did not win on the third move. Note that different sequences of moves can result in the same configuration. After Odd's third

010 000 032	4o9oW 4c9fW 4d9oC 4e9oW 4f9cW 4g9oC 6o7oW 6c7fW 6d9gC
	6e7oW 6f7cW 6g9dC 8o5eW 8c5fW 8d5oC 8e5oW 8f5cW 8g5oC
010 000 074	2o9eW 2c9fW 2d9oC 2e9oW 2f9cW 2g9oC 6o5eW 6c5fW 6d5oC
	6e5oW 6f5cW 6g5oC 8o3eW 8c3fW 8d9gC 8e3oW 8f3cW 8g9dC
010 000 096	2o5eW 2c5eW 2d5eW 2o7oW 2f5oW 2g5eW 4o5eW 4c5eW 4d5eW
	4e5oW 4f5eW 4g5eW 8o5eW 8c5eW 8q5oW 8e7oC 8f5eW 8g5eW
010 000 098	2o5eW 2c5eW 2d5eW 2e5oW 2f5eW 2g5oW 4o5eW 4c5eW 4d5eW
	4o3oW 4f5eW 4g5eW 6o5eW 6c5eW 6d5eW 6e3oC 6f5eW 6g5eW
010 009 600	2o7dW 2c7eW 2d7oW 2e7cW 2h71W 217hW 4o5dW 4c5eW 4d5oW
	4e5cW 4h51W 415hW 8o3dC 8c3dC 8d3cC 8e3cC 8h31C 813hC
010 009 800	2o5dW 2c5eW 2d5oW 2e5cW 2h51W 215hW 4o3dW 4c3eW 4d3oW
	4e3cW 4h31W 413nW 6o7dC 6c7dC 6d7cC 6e7cC 6h71C 617hC
010 030 080	2o5fC 2c5oC 2d5oC 2f5oC 2g51W 215gW 4o7fC 4c7oC 4d71C
	4f7oC 4g71C 417oC 6o91C 6c7oC 6d91C 6f9oC 6g91C 619oC
010 060 090	2o71W 2c7gW 2d7fW 2f7dW 2g7cW 217oW 4o51W 4c5gW 4d5fW
	4f5dW 4g5cW 415oW 8o31C 8c3gC 8d3fC 8f3dC 8g3cC 813oC
010 070 060	2o3fC 2c3oC 2d31C 2f3oC 2g31C 2i3oC 4o5fC 4c5oC 4d5oC
	4f5oC 4g51W 415gW 8o91C 8c3oC 8d91C 8f9oC 8g91C 819oC
010 080 090	2o51W 2c5gW 2d5fW 2f5dW 2g5cW 215oW 4o31W 4c3gW 4d3fW
	4f3dW 4g3cW 413oW 6o71C 6c7gC 6d7fC 6f7dC 6g7cC 617oC
012 000 090	4o5eW 4d5eW 4e7gC 4f5eW 4g5eW 415eW 6o8eW 6d5eW 6e7gW
	6f5eW 6g5eW 615eW 8o5eW 8d5eW 8e5gW 8f5eW 8g5eW 815eW
014 000 090	2o5eW 2d5eW 2e3gC 2f5eW 2g5eW 215eW 6o5oW 6d5eW 6e5gW
	6f5eW 6g5eW 615eW 8o5eW 8d5eW 8e3gW 8f5eW 8g5eW 815eW
015 200 000	4o9gW 4e9oW 4f9oW 4g9oW 4h9oW 419oW 6o7gW 6e9oW 6f9oW
	6g9oW 6h9oW 619oW 8o3gC 8e9oW 8f9oW 8g9oW 8h9oW 819oW

Figure 4(a). Odd's third move.

move, there are five numbers on the board. Thus, Even has only $2 \times 4 = 8$ responses. Using the notation of Figure 4(b). Odd's fourth move is given in Figures 5(a) through 5(d).

Odd's Fifth Move

There is no need to give a table for Odd's fifth move since there are no options at that point. Odd must simply place the remaining number in the sole empty cell on the board. The above strategy is compact enough to be checked by hand, although such a verification would be rather time consuming.

All Initial Winning Moves

In discussing Graham's Game with Scot Morris, the question came up whether the first move given above and the symmetrical moves is the only winning first move, or whether there are other winning first moves. By modifying the search program to analyze all first moves, I was able to see that putting a 1, 3, 7, or 9 in positions b, d, f, and h were winning first moves. Table 1 compares the total number of configurations that were examined to reach this conclusion with the upper bound on the total number of configurations which was derived in Part I.

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015 400 000    2a9gW 2e9aW 2f9aW 2g9aW 2h9aW 219aW 6a7gC 6e9aW 6f9aW
               6g9aW 6h9aW 619aW 8a3gW 8e9aW 8f9aW 8g9aW 8h9aW 819aW
210 000 090    4c5eW 4d5eW 4e71C 4f5eW 4g5eW 415eW 6c5eW 6d7gW 6e71W
               6f5eW 6g7dW 615eW 8c5eW 8d5gW 8e51W 8f5eW 8g5dW 815eW
310 000 200    4c9eW 4d9hC 4e9cW 4f9hC 4h91W 419hW 6c7eW 6d71C 6e7cW
               6f71C 6h71W 617hW 8c5eW 8d51C 8e5cW 8f51C 8h51W 815hW
310 008 030    2c51W 2d5eW 2e5dW 2g51C 2h7dC 215cW 4c71C 4d5hC 4e7dC
               4g71C 4h7gC 417dC 6c71C 6d71C 6e7dC 6g9hC 6h7dC 617dC
310 020 000    4c9gW 4d9fW 4f9dW 4g9cW 4h9gC 419cC 6c7gW 6d7fW 6f7dW
               6g7cW 6h7gC 617hC 8c5gW 8d5fW 8f5dW 8g5cW 8h5gC 815gC
318 000 000    2d51C 2e5gW 2f51W 2g5eW 2h5fC 215fW 4d71C 4e71C 4f71C
               4g7fC 4h7dC 417dC 6d71C 6e91C 6f71C 6g9fC 6h7eC 617dC
410 000 090    2c5oW 2d5eW 2e31C 2f5eW 2g5oW 215eW 6c5oW 6d5gW 6e51W
               6f5oW 6g5dW 615oW 8c5eW 8d3gW 8o31W 8f5oW 8g3dW 815eW
510 002 000    4c91W 4d9cW 4e9cW 4g9cW 4h9cW 419cW 6c71W 6d9cW 6e9cW
               6g9cW 6h9cW 619cW 8c31C 8d9cW 8e9cW 8g9cW 8h9cW 819cW
510 004 000    2c91W 2d9cW 2e9cW 2g9cW 2h9cW 219cW 6c71C 6d9cW 6e9cW
               6g9cW 6h9cW 619cW 8c31W 8d9cW 8e9cW 8g9cW 8h9cW 819cW
510 006 000    2c71W 2d9cW 2e9cW 2g9cW 2h9cW 219cW 4c71C 4d9cW 4e9cW
               4g9cW 4h9cW 419cW 8c3dC 8d9cW 8e9cW 8g9cW 8h9cW 819cW
617 000 000    2d9hC 2e31C 2f9hC 2g3fC 2h9fC 213dC 4d5gW 4e51W 4f5hC
               4g5dW 4h5fC 415eW 8d9hC 8e9gC 8f9hC 8g3fC 8h3fC 819dC
710 000 400    2c9eW 2d9hC 2e9cW 2f9hC 2h91W 219hW 6c5eW 6d5hC 6e5cW
               6f5hC 6h51W 615hW 8c3eW 8d31C 8e3cW 8f31C 8h31W 813hW
710 040 000    2c9gW 2d9fW 2f9dW 2g9cW 2h9gC 219cC 6c5gW 6d5fW 6f5dW
               6g5cW 6h5gC 615gC 8c3gW 8d3fW 8f3dW 8g3cW 8h3gC 813gC
716 000 000    2d31C 2e3gC 2f3gC 2g3fC 2h9dC 213dC 4d5gC 4e5gW 4f51W
               4g5eW 4h5fC 415fW 8d31C 8e91C 8f9gC 8g9fC 8h3eC 813dC
813 000 000    2d5gW 2e51W 2f5hC 2g5dW 2h5fC 215eW 4d9hC 4e71C 4f9hC
               4g7fC 4h9fC 417dC 6d9hC 6e9gC 6f9hC 6g7fC 6h7fC 619dC
910 000 020    4c7dC 4d5cW 4e5cW 4f5cW 4g5cW 415cW 6c7dC 6d5cW 6e5cW
               6f5cW 6g5cW 615cW 8c7dC 8d5cW 8e5cW 8f5cW 8g5cW 815cW
910 000 040    2c3dC 2d5cW 2e5cW 2f5cW 2g5cW 215cW 6c3dC 6d5cW 6e5cW
               6f5cW 6g5cW 615cW 8c3dC 8d5cW 8e5cW 8f5cW 8g5cW 815cW
910 600 000    2c7hC 2d5cW 2f5cW 2g5cW 2h5cW 215cW 4c7hC 4d5cW 4f5cW
               4g5cW 4h5cW 415cW 8c7hC 8e5cW 8f5cW 8g5cW 8h5cW 815cW
910 800 000    2c3hC 2e5cW 2f5cW 2g5cW 2h5cW 215cW 4c3hC 4e5cW 4f5cW
               4g5cW 4h5cW 415cW 6c3hC 6e5cW 6f5cW 6g5cW 6h5cW 615cW
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Figure 4(b). Odd's third move.

Table 1. The Pruning Results for Graham's Game

Size	Configurations	Examined	Percent
1	45	9	20
2	1440	288	20
3	20160	1567	8
4	181440	12529	7
5	907200	40928	5
6	2419200	141909	6
7	3628800	317586	9

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010 009 638    2a7dW 2c7oW 2d7oW 2e5oW 4o5dW 4c5oW 4d5oW 4e5cW
010 009 683    2o7dW 2c7oW 2d7oW 2e7cW 4o5dW 4c5eW 4d5oW 4e5cW
010 009 867    2o5dW 2c5oW 2d5oW 2o5cW 4o3dW 4c3eW 4d3oW 4o3cW
010 009 876    2o5dW 2c5oW 2d5oW 2o5cW 4o3dW 4c3oW 4d3oW 4e5oW
010 030 487    2o9dW 2c5oW 2d9oW 2f5oW 6o5dW 6c5oW 6d5oW 6f5oW
010 030 689    2o7dW 2c7fC 2d7oW 2f7cC 4o5dW 4c5fC 4d5oW 4f5cC
010 070 263    4o9dW 4c5oW 4d9oW 4f5oW 8o5dW 8c5oW 8d5oW 8f5oW
010 070 869    2o5dW 2c5fC 2d5oW 2f5cC 4o3dW 4c3fC 4d3oW 4f3cC
010 270 063    4o9gW 4c5oW 4f5oW 4g9oW 8o5gW 8c5oW 8f5oW 8g5oW
010 368 090    2o71W 2c51W 2g7cW 215cW 4o51W 4c5gW 4g5cW 415oW
010 430 087    2o9gW 2c5oW 2f5oW 2g9oW 6o5gW 6c5oW 6f5oW 6g5oW
010 600 932    4o7cC 4c7fC 4e5fW 4f5oW 8o5eW 8c5fW 8o5oW 8f5cW
010 630 089    2o7gW 2c7fC 2f7cC 2g7oW 4o5gW 4c5fC 4f5cC 4g5oW
010 687 090    2o51W 2c5gW 2g5cW 215oW 4o5gW 4c3gW 4g5oW 413oW
010 786 090    2o51W 2c5gW 2g5cW 215oW 4o31W 4c5iW 4g3cW 415iW
010 800 974    2o3oC 2c3fC 2e5fW 2f5eW 6o5eW 6c5fW 6e5oW 6f5cW
010 863 090    2o5gW 2c7gW 2g5oW 217oW 4o51W 4c5gW 4g5cW 415oW
010 870 069    2o5gW 2c5fC 2f5cC 2g5oW 4o3gW 4c3fC 4f3cC 4g3oW
010 900 632    4o7cC 4c5oW 4e5cW 4f7cC 8o5eW 8c5fW 8e5oW 8f5cW
010 900 874    2o3oC 2c5oW 2e5cW 2f3cC 6o5eW 6c5fW 6e5oW 6f5cW
012 040 790    6o51W 6d5fW 6f5oW 615oW 8o31W 8d3fW 8f3dW 815fW
013 060 890    2o5oW 2d7fW 2f7oW 217oW 4o5iW 4d5fW 4f5dW 415oW
013 089 600    2o7dW 2d7oW 2h71W 217hW 4o5dW 4d5oW 4h51W 415hW
013 809 600    2o7eC 2e7oC 2h71W 217hW 4o5eC 4e5oC 4h51W 415hW
014 020 390    6o71W 6d7fW 6f7oW 615fW 8o51W 8d5fW 8f5dW 815oW
016 080 790    2o51W 2d5fW 2f5oW 215oW 4o31W 4d3fW 4f3dW 415fW
016 709 800    2o5eC 2o5oC 2h51W 215hW 4o3eC 4o3oC 4h31W 413hW
017 069 800    2o5dW 2d5oW 2h51W 215hW 4o3dW 4d3oW 4h31W 413hW
017 080 690    2o51W 2d5fW 2f5oW 215oW 4o5dW 4d3fW 4f3dW 413oW
017 609 800    2o5eC 2e5oC 2h51W 215hW 4o3eC 4o3oC 4h31W 413hW
018 060 390    2o71W 2d7fW 2f7dW 215fW 4o51W 4d5fW 4f5dW 415oW
018 309 600    2o7cC 2c7oC 2h71W 217hW 4o5eC 4e5oC 4h51W 415hW
210 035 000    4c7dW 4d9gW 4g7dW 417dW 6c7dW 6d7gW 6g7dW 617dW
210 040 097    6c5gW 6d5fW 6f5oW 6g5cW 8c3gW 8d3fW 8f3dW 8g5dW
210 073 060    4c5dW 4d9gW 4g5dW 415dW 8c5dW 8d5gW 8g5dW 815dW
310 004 290    6c5eW 6d5eW 6e5dW 615eW 8c5eW 8d5eW 8o5cW 815eW
310 006 207    4c5eW 4d5oW 4e5dW 4h5eW 8c5eW 8d5eW 8e5cW 8h5eW
310 008 205    4c7eW 4d7eW 4e9cW 4h7eW 6c7eW 6d7eW 6e7cW 6h7eW
310 008 407    2c5eW 2d5eW 2e5dW 2h5eW 6c5eW 6d5eW 6e5cW 6h5eW
310 008 690    2c5oW 2d5oW 2e5dW 215eW 4c5eW 4d5eW 4e5cW 415oW
310 008 740    2c5dW 2d5eW 2e5dW 215dW 6c5dW 6d9eC 6e5dW 615dW
310 020 076    4c5fW 4d9fW 4f9dW 4g9cW 8c5gW 8d5fW 8f5dW 8g5cW
310 020 508    4c7dW 4d9fW 4f9dW 4h7dW 6c7dW 6d7fW 6f7dW 6h7dW
310 020 580    4c7dW 4d9fW 4f9dW 417dW 6c7dW 6d7fW 6f7dW 617dW
310 020 760    4c5dW 4d9fW 4f9dW 415dW 8c5dW 8d5fW 8f5dW 815dW
310 020 940    6c71C 6d7fW 6f7dW 617cC 8c51C 8d5fW 8f5dW 815cC
310 060 098    2c5fW 2d7fW 2f7dW 2g7cW 4c5gW 4d5fW 4f5dW 4g5cW
310 070 062    4c9fW 4d9gC 4f9cW 4g9dC 8c5fW 8d5gC 8f5cW 8g5dC
310 072 060    4c91W 4d51W 4g51W 415gW 8c51W 8d51W 8g51W 815cW
310 400 290    6c5eW 6e5fW 6f5eW 615eW 8c5eW 8e5cW 8f5eW 815eW
310 408 050    2c9oW 2e91C 2g9eW 219eW 6c9eW 6e91C 6g9eW 619eW
310 600 207    4c5eW 4e5fW 4f5eW 4h5eW 8c5eW 8e5cW 8f5eW 8h5eW
310 608 007    2c5eW 2e5hC 2g5eW 2h5eW 4c5eW 4e5hC 4g5eW 4h5eW
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Figure 5(a).   Odd's fourth move.

310	708	004	2c5gW	2e5gW	2g9hW	2h9gW	6c5gW	6e5gW	6g5hW	6h5gW
310	708	006	2c5gW	2o5gW	2g5hC	2h5gW	4c5gW	4e5gW	4g5hW	4h5gW
310	708	020	4c5gW	4e5gW	4g91W	419gW	6c5gW	6e5gW	6g91C	615gW
310	708	060	2c5gW	2e5gW	2g51C	215gW	4c5gW	4e5gW	4g51W	415gW
310	748	000	2c5gW	2g9cW	2h5gW	215gW	6c5gW	6g5cW	6h5gW	615gW
310	768	000	2c5gW	2g5cC	2h5gW	215gW	4c5gW	4g5cW	4h5gW	415gW
310	800	205	4c7cW	4o9cW	4f7eW	4h7oW	6c7eW	6e7cW	6f7oW	6h7oW
312	070	060	4d51W	4f91W	4g51W	415gW	8d51W	8f51W	8g51W	815fW
314	008	007	2d5oW	2e5dW	2g5eW	2h5eW	6d5eW	6e5gW	6g5eW	6h5oW
316	008	007	2d5oW	2e5dW	2g5eW	2h5eW	4d5eW	4e5gW	4g5eW	4h5oW
318	004	007	2d9oW	2e9dW	2g5eW	2h5oW	6d5eW	6e5dW	6g5oW	6h5eW
318	005	020	4d9oC	4o9dC	4g91W	419gW	6d7oC	6e7dC	6g71W	617gW
318	006	007	2d5eW	2e5gW	2g5eW	2h5eW	4d5eW	4e5dW	4g5eW	4h5eW
318	007	400	2d9oC	2e9dC	2h91W	219hW	6d5eC	6e5dC	6h51W	615hW
318	009	600	2d7eC	2e7dC	2h71W	217hW	4d5eC	4e5dC	4h51W	415hW
318	040	007	2d9fW	2f9dW	2g9hC	2h9gC	6d5fW	6f5dW	6g5hC	6h5gC
318	060	009	2d7fW	2f7dW	2g7hC	2h7gC	4d5fW	4f5dW	4g5hC	4h5gC
318	070	060	2d51W	2f51W	2g51W	215fW	4d51W	4f51W	4g51W	415gW
318	200	005	4e9fW	4f9eW	4g7eW	4h7eW	6e7fW	6f7eW	6g7eW	6h7eW
318	400	007	2e9fW	2f9eW	2g5eW	2h5eW	6e5fW	6f5eW	6g5eW	6h5eW
318	600	007	2e5gW	2f5eW	2g5eW	2h5eW	4e5fW	4f5eW	4g5eW	4h5eW
318	700	004	2e5gW	2f5gW	2g9hW	2h9gW	6e5gW	6f5gW	6g5hW	6h5gW
318	700	006	2o5gW	2f5gW	2g5eW	2h5gW	4e5gW	4f5gW	4g5hW	4h5gW
318	700	040	2e5gW	2f5gW	2g91W	219gW	6e5gW	6f5gW	6g51W	615gW
319	020	004	6d7fW	6f7dW	6g5hW	6h5gW	8d5fW	8f5dW	8g5hC	8h5gC
410	020	093	6c7gW	6d7fW	6f7dW	6g5dW	8c5gW	8d5fW	8f5dW	8g5cW
410	037	080	2c5dW	2d9gW	2g5dW	215dW	6c5dW	6d5gW	6g5dW	615dW
410	075	060	2c3dW	2d9gW	2g3dW	213dW	8c3dW	8d3gW	8g3dW	813dW
510	000	674	2c9fW	2d9cW	2e9cW	2f9cW	8c3fW	8d9cW	8e9cW	8f9cW
510	000	832	4c9fW	4d9cW	4e9cW	4f9cW	6c7fW	6d9cW	6e9cW	6f9cW
510	032	080	4c91W	4d9cW	4g9cW	419cW	6c71W	6d9cW	6g9cW	619cW
510	074	060	2c91W	2d9cW	2g9cW	219cW	8c31W	8d9cW	8g9cW	819cW
510	230	080	4c71W	4f9cW	4g9cW	419cW	6c71W	6f9cW	6g9cW	619cW
510	470	060	2c31W	2f9cW	2g9cW	219cW	8c31W	8f9cW	8g9cW	819cW
510	600	074	2c9fW	2e9cW	2f9cW	2g9cW	8c3fW	8e9cW	8f9cW	8g9cW
510	800	032	4c9fW	4e9cW	4f9cW	4g9cW	6c7fW	6e9cW	6f9cW	6g9cW
512	030	080	4d71W	4f91W	4g71W	419fW	6d71W	6f71W	6g71W	617fW
514	006	007	2d3eW	2e9gW	2g3eW	2h3eW	8d3eW	8e3gW	8g3eW	8h3eW
514	070	060	2d31W	2f91W	2g31W	219fW	8d31W	8f31W	8g31W	813fW
516	004	007	2d9eW	2e9dW	2g3eW	2h3eW	8d3eW	8e3dW	8g3eW	8h3eW
518	002	003	4d9eW	4e9dW	4g7eW	4h7eW	6d7oW	6e7dW	6g7eW	6h7eW
518	306	000	2e7gW	2g7eC	2h7gW	217gW	4e7gW	4g7eC	4h7gW	417gW
610	030	089	2c7fC	2d7gW	2f7cC	2g7dW	4c5fC	4d5gW	4f5cC	4g5dW
610	080	097	2c5gW	2d5fW	2f5dW	2g5cW	4c3gW	4d3fW	4f3dW	4g5dW
610	709	801	2c5oW	2e5cW	2h51W	215hW	4c3eW	4e51W	4h31W	413hW
615	400	700	2e9fW	2f9oW	2h3eW	213eW	8e3fW	8f3eW	8h3eW	813eW
617	002	090	4d5gW	4e51W	4g5dW	415eW	8d5oW	8e5dW	8g5eW	815eW
617	003	080	2d51W	2e51W	2g51W	215gW	4d5gW	4e51W	4g5dW	415eW
617	003	200	4d51W	4e51W	4h91W	419hW	8d51W	8e51W	8h51W	815hW
617	003	800	2d51W	2o51W	2h51W	215hW	4d51W	4e51W	4h51W	415eW
617	004	050	2d9oW	2e9dW	2g9oW	219eW	8d3eW	8e3dW	8g9eW	819eW
617	005	040	2d31W	2e31W	2g91W	219gW	8d31W	8e31W	8g31W	813gW
617	008	090	2d5eW	2e5dW	2g5eW	215eW	4d3eW	4e3dW	4g5dW	415eW

Figure 5(b). Odd's fourth move.

617	009	020	4d5gW	4o51W	4g5dW	415eW	8d3gC	8e31C	8g51W	815gW
617	020	003	4d9fW	4f9dW	4g5dW	4h5fW	8d5fW	8f5dW	8g5fW	8h5fW
617	080	900	2d5fW	2f5dW	2h51C	215hC	4d3fW	4f3dW	4h31C	413hC
617	200	090	4e51W	4f5eW	4g5eW	415eW	8e5fW	8f5eW	8g5eW	815eW
617	300	002	4o9fC	4f9eC	4g9hW	4h9gW	8e5fC	8f5eC	8g5hW	8h5gW
617	800	090	2o5fW	2f5eW	2g5eW	215eW	4e3fW	4f3eW	4g5eW	415eW
617	900	008	2o5fC	2f5oC	2g5hW	2h5gW	4o3fC	4f3oC	4g3hW	4h3gW
710	002	490	6c5oW	6d5oW	6e5cW	615eW	8c5oW	8d5eW	8e5dW	815oW
710	006	450	2c9eW	2d9eW	2e9cW	219eW	8c9eW	8d9eW	8e3cW	819eW
710	008	403	2c5oW	2d5eW	2e5dW	2h5oW	6c5oW	6d5oW	6e5cW	6h5eW
710	030	004	2c9fW	2d9gC	2f9cW	2g9dC	6c5fW	6d5gC	6f5cW	6g5dC
710	034	080	2c91W	2d51W	2g51W	215gW	6c51W	6d51W	6g51W	615cW
710	040	308	2c5dW	2d9fW	2f9dW	2h5dW	6c5dW	6d5fW	6f5dW	6h5dW
710	040	380	2c5dW	2d9fW	2f9dW	215dW	6c5dW	6d5fW	6f5dW	615dW
710	040	506	2c3dW	2d9fW	2f9dW	2h3dW	8c3dW	8d3fW	8f3dW	8h3dW
710	040	560	2c3dW	2d9fW	2f9dW	213dW	8c3dW	8d3fW	8f3dW	813dW
710	040	920	6c51C	6d5fW	6f5dW	615cC	8c31C	8d3fW	8f5uW	813cC
710	080	096	2c5gW	2d5fW	2f5dW	2g5cW	4c5fW	4d3fW	4f3dW	4g3cW
710	200	490	6c5oW	6o5cW	6f5eW	615eW	8c5oW	8e5fW	8f5eW	815eW
710	600	450	2c9oW	2o9cW	2f9eW	219eW	8c9eW	8o3cW	8f9eW	819eW
710	800	403	2c5oW	2o5fW	2f5eW	2h5eW	6c5eW	6o5cW	6f5eW	6h5eW
714	030	080	2d51W	2f91W	2g51W	215gW	6d51W	6f51W	6g51W	615fW
716	002	300	4d9oW	4e9dW	4h5dW	415dW	8d5eW	8e5dW	8h5dW	815dW
716	003	200	4d9eC	4e9dC	4h91W	419hW	8d5eC	8e5dC	8h51W	815hW
716	005	040	2d9eC	2e9dC	2g91W	219gW	8d3eC	8o3dC	8g31W	813gW
716	008	900	2d5eW	2e5dW	2h51C	215hC	4d3eW	4o3dW	4h31C	413hC
716	009	800	2d5oC	2e5dC	2h51W	215hW	4d3eC	4o3dC	4h31W	413hW
716	020	300	4d9fW	4f9dW	4h5dW	415dW	8d5fW	8f5dW	8h5dW	815dW
716	030	080	2d51W	2f51W	2g51W	215gW	4d51W	4f51W	4g51W	415fW
716	080	009	2d5fW	2f5dW	2g5hC	2h5gC	4d3fW	4f3dW	4g3hC	4h3gC
716	200	003	4o9fW	4f9eW	4g5eW	4h5oW	8e5fW	8f5oW	8g5eW	8h5eW
716	300	002	4e5gW	4f5gW	4g9hW	4h9gW	8e5gW	8f5gW	8g5hW	8h5gW
716	300	008	2o5gW	2f5gW	2g5hW	2h5gW	4e5gW	4f5gW	4g5eW	4h5gW
716	400	500	2o9fW	2f9oW	2h91C	219hC	8o3fW	8f3eW	8h31C	813hC
716	800	003	2o5fW	2f5eW	2g5eW	2h5eW	4e5gW	4f5eW	4g5eW	4h5eW
716	900	020	4e5gW	4f51W	4g5eW	415fW	8e3gC	8f31C	8g51W	815gW
719	040	002	6d5fW	6f5dW	6g5hC	6h5gC	8d3fW	8f3dW	8g5hW	8h5gW
810	060	093	2c7gW	2d7fW	2f7dW	2g5dW	4c5gW	4d5fW	4f5dW	4g5cW
810	070	069	2c5fC	2d5gW	2f5cC	2g5dW	4c3fC	4d3gW	4f3cC	4g3dW
810	309	600	2c7oW	2o51W	2h71W	217hW	4c5oW	4o5cW	4h51W	415hW
813	002	050	4d9oW	4e9dW	4g9oW	419oW	6d7oW	6o7dW	6g9oW	619oW
813	004	090	2d5gW	2o51W	2g5dW	215oW	6d5oW	6o5dW	6g5oW	615oW
813	005	020	4d71W	4o71W	4g91W	419gW	6d71W	6o71W	6g71W	617gW
813	006	090	2d7oW	2o7dW	2g5dW	215oW	4d5oW	4o5dW	4g5oW	415oW
813	007	060	2d5gW	2o51W	2g5dW	215oW	4d51W	4o51W	4g51W	415gW
813	007	400	2d51W	2e51W	2h91W	219hW	6d51W	6o51W	6h51W	615hW
813	007	600	2d51W	2e51W	2h51W	215oW	4d51W	4o51W	4h51W	415hW
813	009	040	2d5gW	2o51W	2g5dW	215oW	6d7gC	6o71C	6g51W	615gW
813	040	007	2d9fW	2f9dW	2g5dW	2h5fW	6d5fW	6f5dW	6g5fW	6h5fW
813	060	900	2d7fW	2f7dW	2h71C	217hC	4d5fW	4f5dW	4h51C	415hC
813	400	090	2o51W	2f5oW	2g5oW	215oW	6e5fW	6f5oW	6g5oW	615oW
813	600	090	2o7fW	2f7eW	2g5eW	215oW	4e5fW	4f5eW	4g5oW	415eW
813	700	004	2o9fC	2f9oC	2g9hW	2h9gW	6e5fC	6f5eC	6g5hW	6h5gW

Figure 5(c). Odd's fourth move.

```
813 900 006    2c7fC 2f7cC 2g7hW 2h7gW 4o5fC 4f5cC 4g5hW 4h5gW
815 200 300    4c9fW 4f9cW 4h7cW 4i7cW 6o7fW 6f7eW 6h7eW 6i7cW
910 000 274    6c5fW 6d5cW 6o5cW 6f5cW 8c3fW 8d5cW 8e5cW 8f5cW
910 000 432    6c7fW 6d5cW 6o5cW 6f5cW 8c5fW 8d5cW 8e5cW 8f5cW
910 030 086    2c7fW 2d5cW 2f5cW 2g5cW 4c5fW 4d5cW 4f5cW 4g5cW
910 036 080    2c7iW 2d5cW 2g5cW 2i5cW 4c5iW 4d5cW 4g5cW 4i5cW
910 070 068    2c5fW 2d5cW 2f5cW 2g5cW 4c3fW 4d5cW 4f5cW 4g5cW
910 078 060    2c5iW 2d5cW 2g5cW 2i5cW 4c3iW 4d5cW 4g5cW 4i5cW
910 200 074    6c5fW 6o5cW 6f5cW 6g5cW 8c3fW 8e5cW 8f5cW 8g5cW
910 400 032    6c7fW 6o5cW 6f5cW 6g5cW 8c5fW 8o5cW 8f5cW 8g5cW
912 300 040    6o7gW 6f7iW 6g5iW 6i5gW 8e5gW 8f5iW 8g5oW 8i5iW
912 600 070    4o5fW 4f5oW 4g3oC 413fC 8e5gW 8f5iW 8g5oW 8i5fW
912 800 030    4o5gC 4f5iC 4g5eC 415fC 6e7gW 6f7iW 6g7eW 6i7fW
914 600 070    2c5gC 2f5iC 2g5eC 215fC 8e3gW 8f3iW 8g3eW 8i3fW
914 700 020    6c5gW 6f5iW 6g5eW 615fW 8e3gW 8f3iW 8g5iW 8i5gW
914 800 030    2o5fW 2f5oW 2g7oC 217fC 6e5gW 6f5iW 6g5oW 6i5fW
916 300 040    2o7gW 2f7iW 2g7oW 217fW 8e5gC 8f5iC 8g5oC 8i5fC
916 700 020    4o5gW 4f5iW 4g5oW 415fW 8e3gC 8f3iC 8g5iW 8i5gW
916 800 030    2o5fW 2f5eW 2g7oW 217fW 4o5gW 4f5iW 4g5oW 415fW
918 300 040    2o5gW 2f5iW 2g5oW 215fW 6e7gC 6f7iC 6g5iW 615gW
918 600 070    2o5gW 2f5iW 2g5oW 215fW 4o5fW 4f5oW 4g3oW 413fW
918 700 020    4o3gW 4f3iW 4g3oW 413fW 6e5gC 6f5iC 6g5oC 615fC
```

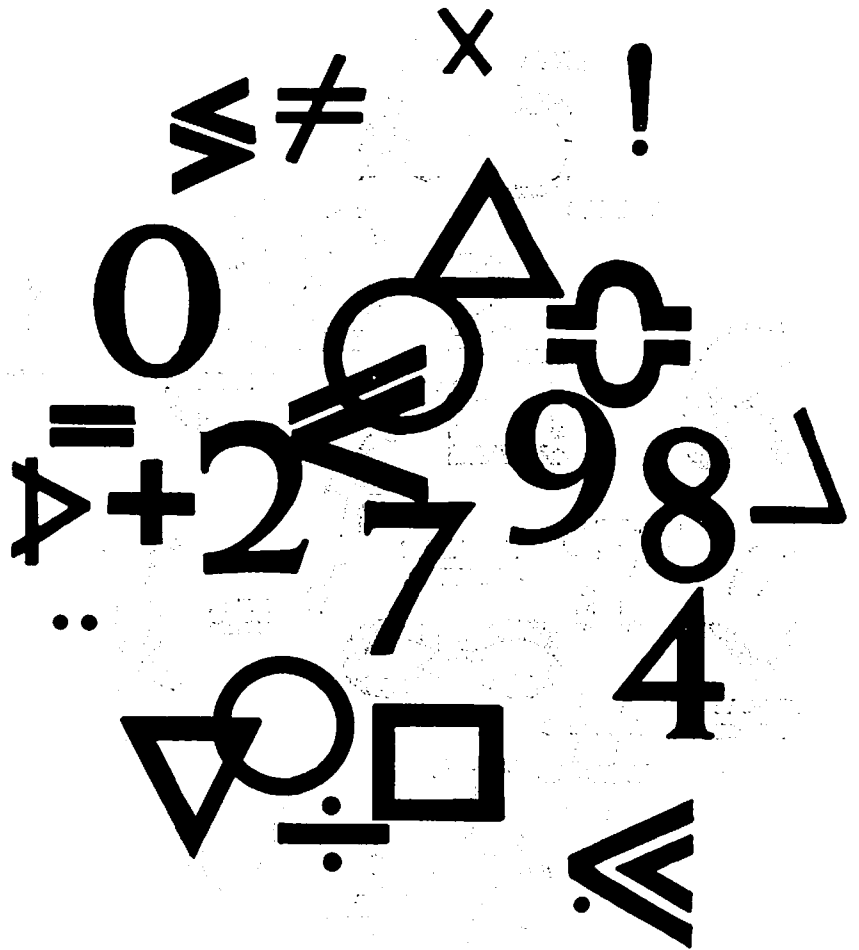Figure 5(d). Odd's fourth move.

Acknowledgments

Note

William Conklin has noted that the example shown in Figure 5 of [1] is correct only if Odd places 9 in position X, otherwise Odd can get a line of 15 by placing 9 in the lower right corner. Even's correct strategy is the following: If Odd blocks with a 9, 7, 3, or 1, Even responds with a 4, 6, 8, or 8 in the center. Whenever Odd's first move uses 5, Even can always threaten to win by using 2 appropriately. If Odd blocks using 9, 7, 3, or 1, Even can always force a win by responding with a 4, 6, 8, or 8.

Reference

1. G. Markowsky, Numerical Tic-Tac-Toe—I, *Journal of Recreational Mathematics,* 22:2, pp. 114-123, 1990.

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