Syndrome-Testability Can be Achieved by Circuit Modification

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***Abstract*—In [1] and [2] Savir developed many facets of syndrome-testing (checking the number of minterms realized by a circuit against the number realized by a fault-free version of that circuit) and presented evidence showing that syndrome-testing can be used in many practical circuits to detect all single faults. In some cases, where syndrome-testing did not detect all single stuck-at-faults, Savir showed that by the addition of a small number of additional "control" inputs and gates one would get a function which is syndrome-testable for all single stuck-at faults, and yet which realizes the original function when the "control" inputs are fed appropriate values. However, he left open the question of whether one could always modify a circuit to achieve syndrome-testability. In this correspondence we show that a combinatorial circuit can always be modified to produce a single-fault, syndrome-testable circuit.**

***Index Terms*—Circuit modification, stuck-at-faults, syndrome testability.**

Syndrome testing offers the possibility of simple and thorough testing of circuits and even the possibility of self-testing. Unfortunately, a circuit need not be syndrome-testable. Savir [1], [2] showed that many circuits of "practical" interest are either syndrome-testable or can be made syndrome-testable by the addition of a small number of additional control lines. This correspondence shows that a circuit can always be made syndrome-testable by adding "enough" control lines. Since there does not exist a handy and useful characterization of circuits of "practical" interest, the approach used here is based on no particular property of the circuit being analyzed, and consequently it is not surprising that the technique we use to prove the main results is not of practical interest. The chief point of this correspondence is to demonstrate that syndrome-testability can always be achieved. It is my hope that knowledge of this fact will spur people to analyze further the problem of modifying "practical" circuits to achieve syndrome-testability. The reader is urged to consult Savir's papers for basic information on syndrome.

Definition: Let $x_1, \cdots, x_n$ be Boolean variables and F a Boolean function in some subset of these variables. Let K be the number of assignments of values to the variables which give F the value of 1. Then the *syndrome* of F, $S(F)$ is defined to be the quantity $K/2^n$. Finally, the *cosyndrome* of F, $C(F)$ is defined to be the quantity $1 - S(F)$. □

Fig. 1 is a schematic representation of an OR gate in a typical combinatorial circuit. Fig. 2 shows how it can be modified by adding additional AND gates and various numbers of additional control variables. For a NOR gate, it is clear that the solution would be the same as for the OR gate since the syndrome of NOR circuit is simply the cosyndrome of the corresponding OR circuit. For AND and NAND circuits we use additional OR gates, rather than AND gates.

Theorem 1: The procedure described above can always be tailored so that the circuit (of Fig. 2) ending at OUT is single-fault syndrome-testable assuming that each of the circuits ending at the l_i are single-fault syndrome-testable.

Proof: The remarks made above show that the result for the OR case would imply the result for the NOR case. Furthermore, by substituting cosyndrome for syndrome in the argument below one gets the result in the AND case and hence the NAND case. Thus, we only give the proof in the OR case, i.e., the case of Figs. 1 and 2.

Let $B_i(*B_i)$ denote the function realized by the circuits feeding line $l_i(*l_i)$ and let $B_i^f(*B_i^f)$ denote the same circuit having a single fault f somewhere in it. It is also reasonable to assume $B_i \not\equiv 0, 1$ to avoid discussing trivialities. Let $B(*B)$ denote the function realized at OUT in Fig. 1 (2) and $B^f(*B^f)$ the function which results from a single fault f somewhere in it.

Let $\delta = \max \{\delta_i\}$, where δ_i is the number of variables which eventually feed into line l_i. Finally, let $k_1 < k_2 < \cdots < k_m$ be chosen so that $2^{-(k_i+\delta)} > 10m2^{-k_{i+1}}$. For consistency let $k_0 = 0$.

The proof is inductive in nature and begins by noticing that simple gates are syndrome-testable. Next, we observe that any single stuck-at fault in OUT or any $*l_i$ is syndrome-testable. By induction we assume that $S(B_i^f) \neq S(B_i)$ for all single faults occurring in the circuit feeding l_i. It immediately follows that $S(*B_i^f) \neq S(*B_i)$ for all single faults occurring in the circuit feeding $*l_i$, since either the fault occurs in B_i, whence $S(*B_i^f) = S(B_i^f)/2^{k_i}$ and $S(*B_i) = S(B_i)/2^{k_i}$, or the fault occurs in some c_{ij} or $*l_i$ and either $S(*B_i^f) = 0, 1$ or $S(B_i)/2^{k_i-1}$, while $S(*B_i) = S(B_i)/2^{k_i} \neq 0, 1$.

In order to prove that $S(*B) \neq S(*B^f)$ for all f we proceed as follows. Let i_0 be such that $*B_i^f \equiv *B_i$ for $1 \leq i < i_0$, but $*B_{i_0}^f \not\equiv *B_{i_0}$.

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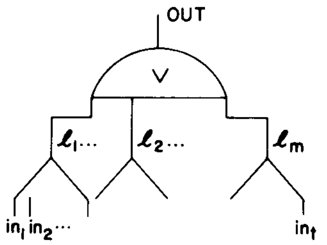

Fig. 1. A schematic representation of part of a combinational circuit.

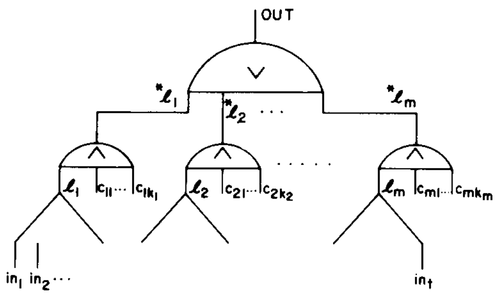

Fig. 2. A circuit modified to ensure syndrome testability.

Since at most $k_{i_0} + \delta$ lines feed l_{i_0}, we must have $|S(*B^f_{i0}) - S(*B_{i_0})| \geq 2^{-(k_{i0}+\delta)}$ because the syndromes of these two subcircuits have denominators $\leq 2^{-(k_{i0}+\delta)}$ and numerators which differ by at least 1. We consider two cases as follows.

1) $S(*B^f_{i0}) \geq S(*B_{i_0}) + 2^{-(k_{i0}+\delta)}$: In this case since the syndrome of a disjunction of circuits is not greater than the sum of the syndromes of the subcircuits

$$S(*B) \leq S(\bigvee_{i \leq i_0} *B_i) + S(*B_{i_0})$$
$$+ S(\bigvee_{i > i_0} *B_i) \leq S(A) + S(*B_{i_0}) + m2^{-k_{i0+1}}$$

where $A = \bigvee_{i \leq i_0} *B_i$, since for $i > i_0$ $S(*B_i) \leq 2^{-k_i} \leq 2^{-k_{i0+1}}$.

However, $m2^{-k_{i0+1}} \leq (1/10)2^{-(k_{i0}+\delta)}$ because of the way we selected the k_i.

Conversely, $S(*B^f) \geq S(A) + S(*B^f_{i0}) - S(A \wedge *B^f_{i0})$, since the quantity on the right-hand side is exactly the syndrome of $S(A \vee *B^f_{i0})$, and is based on the fact that the number of elements in the union of two sets is the sum of the number of elements in each set minus the number in their intersection.

Now we use the fact that the syndrome of a disjunction of circuits is not greater than the sum of the syndromes of the circuits to obtain

$$S(A \wedge *B^f_{i0}) \leq \sum_{i=1}^{i0} S(*B_i \wedge *B^f_{i0}) \leq m2^{-k_1}2^{-(k_{i0}-1)}$$
$$\leq (1/5)2^{-(k_{i0}+\delta)}.$$

The fact that each $S(*B_i \wedge *B^f_{i0}) \leq 2^{-k_i}2^{-(k_{i0}-1)} \leq 2^{-k_1}2^{-(k_{i0}-1)}$ follows since f is a single fault, we have at least $k_i + k_{i_0}$ *independent* control lines and k_1 is the smallest of the k_i's. Note that we used the fact that f was a single-fault to observe that at most one of the control lines $c_{i_01}, \cdots, c_{i_0k_{i0}}$ is stuck-at 1 (a control line stuck-at 0 is equivalent to a $*l_i$ stuck-at 0). Thus, we have $S(*B) \leq S(A) + S(*B_{i_0}) + (1/10)2^{-(k_{i0}+\delta)}$, but $S(*B^f) \geq S(A) + S(*B_{i_0}) + (4/5)2^{-(k_{i0}+\delta)}$. Thus, $S(*B^f) \neq S(*B)$.

2) $S(*B_{i_0}) \geq S(*B^f_{i0}) + 2^{-(k_{i0}+\delta)}$: In this case arguing as above we get that

$$S(*B) \geq S(A) + S(*B^f_{i0}) + (9/10)2^{-(k_{i0}+\delta)}$$

and

$$S(*B^f) \leq S(A) + S(*B^f_{i0}) + m2^{-(k_{i0+1}-1)}$$
$$\leq S(A) + S(*B^f_{i0}) + (1/5)2^{-(k_{i0}+\delta)}$$

where we use the fact that f is a single-fault to conclude that $S(*B^f_i) \leq 2^{-(k_i-1)}$. Thus, again $S(*B^f) \neq S(*B)$. □

The above proof actually proves a lot more than Theorem 1. This stronger result is given in Theorem 2.

Theorem 2: The procedure of Theorem 1 produces a circuit with the following properties:

1) if f and g are single faults such that for some i, $S(*B^f_i) \neq S(*B^g_i)$, then $S(*B^f) \neq S(*B^g)$;

2) if f is any multiple fault involving only the l_i and any lines feeding the l_i such that for some i, $S(B^f_i) \neq S(B_i)$, then $S(*B^f) \neq S(*B)$.

Proof: Both 1) and 2) above are proved essentially the same way that Theorem 1 is proved. For 1) we pick i_0 minimal with the property that $S(*B^f_{i0}) \neq S(*B^g_{i0})$. As before, these two values must differ by at least $2^{-(k_{i0}+\delta)}$ and the rest of the argument in Theorem 1 goes through essentially unchanged (1/10 and 9/10 become 1/5 and 4/5, respectively). For 2) the argument is similar since all the bounds in Theorem 1 are based on the number of control lines and we only use the fact that $S(B^f_i)$ is between 1 and 0, which is true regardless of whether f is a single or multiple fault. Indeed, the only time we use the assumption that f is a single fault is in limiting its effect on the control lines. □

Theorem 2 shows that we have a limited ability to diagnose single faults and detect multiple faults. Diagnosing all single faults is not something one would expect from syndrome testing since one cannot do it for simple ∨ and ∧ gates. In particular, the modifications described above will not produce circuits in which every single fault is syndrome diagnosable. Diagnosing multiple faults is also not feasible for two additional reasons: 1) the large amount of storage necessary to store the syndrome values for each multiple fault defeats the whole purpose of syndrome testing; 2) in modifying a circuit by adding control lines to achieve a syndrome-testable design there is always a multiple fault in the control lines which yields the original non-syndrome-testable circuit.

The question of multiple fault detection is not fully resolved. The above argument will handle multiple faults that are not "too bad," i.e., where the number of faults allowed is bounded by a suitable function of the number of lines. To detect all multiple faults would seem to require an even more complicated analysis and more control lines. For this reason we have chosen not to pursue this problem further.

ACKNOWLEDGMENT

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REFERENCES

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