Name: _____

- This exam is open book, open notes, but there can be no sharing of any material.
- You can use the Internet, but only as a library.
- If you are not sure if something is allowed, please ask.
- If you use any material other than the assigned readings and your own prior work, you must prominently indicate the source in your answers.
- There are 6 questions (including one extra-credit question) on 6 pages.
- You have *40 minutes* to earn *40 points* (plus 10 extra-credit points). You may wish to use this correspondence to plan your time.

1. (1 pt.) Write your name in the space provided above. Answer the rest of this exam based on on the *qsort* paper.[1]

2. (10 pts.) Suppose we have a list L of names of the form *(lastname, firstname)* or, briefly, (l, f). Let $S(x, k)$ denote the result of sorting the list x in order of key k (where k is l or f for this question) using a *stable* sorting method. Similarly, let $T(x, k)$ denote the result of sorting x in order of k using a *non-stable* sorting method. We wish to produce a list of names sorted by lastnames, with names that have the same lastname appearing together as a group and ordered by firstnames within that group (i.e., the order names are typically found in a phone book). Which, if any, of the following actions are *guaranteed* to produce the desired result?

   (a) $S(T(L, l), f)$

   (b) $T(S(L, l), f)$

   (c) $S(T(L, f), l)$

   (d) $T(S(L, f), l)$

   Justify your answer. [Hint: Try a small example.]

[1] Jon L. Bentley and M. Douglas McIlroy, "Engineering a Sort Function," *Software–Practice and Experience* 23/11 (1993).

[additional space for answering the question on the previous page]

3. (9 pts.) Consider the `iisort` program on page 1250. How many comparisons (of array elements) does it make when the input consists of n elements in sorted order (e.g., $1, 2, 3, 4, 5$)? Express your answer as a function of n in as compact a form as possible. *Justify your answer.* [Hint: Try a small example.]

4. (10 pts.) How many array-element comparisons does `iisort` make when the input consists of n elements in *reverse* sorted order (e.g., 5,4,3,2,1)? Express your answer as a function of n in as compact a form as possible. *Justify your answer.* [Hint: Try a small example.]

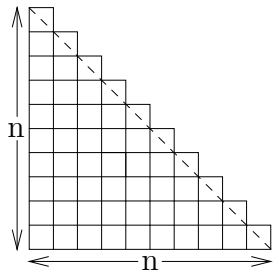
5. (10 pts) Summarize the actions of the `iqsort0` program (Program 2, page 1252) on the input

   2 7 1 8 2 8 1 8 2 8

   by completing the following table. Row r of the table represents the state of the variables i and j, and of the array, at the end of the r'th iteration of the for loop in `iqsort0`. Row 0 represents the state at the beginning of the first iteration. Row 10 represents the state just before the first recursive call to `iqsort0` (i.e., just before the second-to-last line of the program).

row#	i	j	array by index									
			0	1	2	3	4	5	6	7	8	9
0	1	0	2	7	1	8	2	8	1	8	2	8
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												

6. (10 pts) **(Extra Credit)** Consider the following layout of tiles on the floor of an $n \times n$ corner of a room. Each tile measures 1×1. All units are the same, say feet. The diagram illustrates the case for $n = 10$, but your answers should be expressed using n for the general case, not tailored to the example.



Compute the area covered by the tiles in two ways, showing your work. [Hint: Read the entire question first.]

(a) Count the number of tiles and multiply that number by the area of each tile.

(b) Add the area of the large triangle below the dashed line to the areas of all the small triangles above the dashed line.

Since both ways must yield the same result, what commonly used arithmetic identity have we just proved?[2] [Hint: We have used this identity in class several times.]

[2]This proof is due to Ian Richards, "Proof without Words: Sum of Integers," *Mathematics Magazine* 57/2 (1984).