The main goal of today's exercise is to practice applying some of the ideas on graphs and permutation generation that we have studied earlier.

Reminders:

1. Write carefully; this material does count for class participation.
2. Note the requirements for Capstone events on April 30th.
3. Meet your advisors often.
4. Invite friends and family liberally to the Capstone presentations.

1. List the members of your group below, underlining your name.

2. Indicate how many people you are inviting to, and expect to attend, the Capstone presentations on April 30th. Include yourself. Your best estimate is fine; no guarantee is required. ______

   List the full names of the above people, and indicate the relationship of each with you (e.g., mother, friend, spouse, great uncle, paid shill).

3. For an integer $n > 1$, let V_n be the set of $(n-2)$-character strings $\{x_1 x_2 \ldots x_{n-2} \mid x_i \in \{1, 2, \ldots, n\}\}$. List V_n for $n = 2, 3, 4$.

4. For an integer $n > 1$, define a digraph $Q_n = (V_n, E_n)$ where the set of vertices V_n defined in Question 3 and the set of edges $E_n = \{(u, v) \mid u, v \in V_n$ with $u = x_1x_2x_3\cdots x_{n-2},\ \ v = x_2x_3\cdots x_{n-2}x_{n-1}$, where $x_i \neq x_j$ for $i \neq j\}$. Depict Q_n for $n = 2, 3, 4$.

5. Do the graphs Q_2, Q_3, and Q_4 of Question 4 have Eulerian paths? For each graph, exhibit an Eulerian path or explain why no such path exists.

   Recall that an Eulerian path in a digraph is a directed path that traverses each edge exactly once. A digraph with such a path is called Eulerian.

6. Prove or disprove: The graphs Q_n of Question 4 are Eulerian for all $n > 1$.

7. The destination of every edge in the digraph Q_n (of Question 4) is a string of length $n-2$ whose $(n-3)$-character prefix is a suffix of that edge's source. Therefore, we may encode an Eulerian path by a string with the first $n-2$ characters representing the initial vertex and with each following character representing an edge on the path, in order.

   Encode all the Eulerian paths of Question 5 (if any) by the strings described above.

   What is the length of a string that encodes an Eulerian path of Q_n, as a function of n?

8. For each string listed for graph Q_n in Question 7, list here all $(n-1)$-character substrings generated by sliding a window of length $n-1$ along the string.

   How many such substrings are generated for Q_n, as a function of n?

9. Augment each substring listed in Question 8 (for a graph Q_n) by suffixing it with the single element of $\{1, 2, \ldots, n\}$ that it does not already contain, yielding strings of length n.

   Note and explain the most interesting observations you can make based on the above.