Name: __________________________________________

- COS 480 students should answer non-⋆ questions; optional ⋆ questions are for extra credit.
- COS 580 students should answer all questions, including ⋆ questions.

1. (1 pt.) Write your name in the space provided above.

2. (19 pts.) Consider a database with the following familiar schema, as discussed in class.
   (Ask for clarifications if the semantics are unclear.)
   
   \[
   \begin{align*}
   \text{Students}(\text{sid}, \text{name}, \text{year}, \text{major}) \\
   \text{Courses}(\text{cid}, \text{title}, \text{ta}) \\
   \text{Enrolls}(\text{sid}, \text{cid}, \text{credits})
   \end{align*}
   \]

   Consider the following constraints.

   (a) Values for credits must be non-negative integers.
   (b) Every student ID appearing in the Enrolls table must also appear in the Students table.
   (c) Values for the year must be two-character codes.
   (d) Students cannot TA courses in which they are enrolled.
   (e) Students with major ‘COS’ cannot be enrolled in any class for fewer than 3 credits.
   (f) Students with major ‘COS’ cannot have ‘NN’ for the year attribute.

   Provide standard SQL statements that create the tables by selecting the most appropriate types, keeping the constraints in mind. Justify your selections and explain briefly how your selections assist in maintaining consistency. Provide standard SQL statements to declare the constraints listed above. Use the simplest (least powerful) possible SQL constraint type for each.
3. (15 pts.) The division operator of relational algebra takes operands $R(A, B, C, D)$ and $S(A, B)$, and produces the quotient:

$$R \div S \equiv \{(c, d) \mid \forall (a, b) \in S : (a, b, c, d) \in R\}$$

Express $R \div S$ using a relational algebra expression composed of no operators other than selection, projection, cross product, union, and difference. Prove that your expression is equivalent to the above definition.
[additional space for answering the earlier question]
4. (10 pts.) Given a SQL database with tables \( R(a,b,c,d) \) and \( S(a,b) \), provide a standard SQL statement to declare the constraint that \( R \div S \) must be empty. (The tables are assumed to exist and to be populated.)
5. (10 pts.) Define an extended bag algebra operator $\eta$ that, intuitively, is similar to the division operator of Question 3 but that instead of “for all tuples in $S$” uses “for at least half the tuples in $S$.” In more detail, $R\eta S$ contains tuples $(c, d)$ such that there are at least $\lceil |S| / 2 \rceil$ tuples $(a, b) \in S$ such that $(a, b, c, d) \in R$. Duplicates should be preserved from $R$ to $R\eta S$ and counted separately in $S$.

*Prove or disprove:* $R\eta S$ may be expressed using standard extended bag algebra (as defined in the textbook and in class).
6. (5 pts.) * Repeat Question 4 replacing the operator $\div$ by the operator $\eta$ of Question 5.