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  - 1. List the members of your group below. Underline your name.
  - 2. Define sorting and comparison sorting.

3. Name a well-known comparison-sorting algorithm and another sorting algorithm that is not a comparison-sort.

4. Define a *permutation* of a collection of objects. List all permutations of the collection  $\{1, 1, 3, 5, 5\}.$ 

- 5. For an integer n > 1, let  $V_n$  be the set of (n-2)-character strings  $\{x_1x_2...x_{n-2} \mid x_i \in \{1, 2, ..., n\}$  and  $x_i \neq x_j$  unless  $i = j\}$ .
  - (a) List  $V_n$  for n = 2, 3, 4.

  - (b) What is the cardinality of V<sub>n</sub>, as a function of n?
    (c) Provide an alternate, equivalent (perhaps simpler) definition of V<sub>n</sub>.

- 6. For an integer n > 1, define a digraph  $Q_n = (V_n, E_n)$  where the set of vertices  $V_n$  defined in Question 5 and the set of edges  $E_n = \{(u, v) \mid u, v \in V_n \text{ with } u = x_1 x_2 x_3 \cdots x_{n-2}, v = x_2 x_3 \cdots x_{n-2} x_{n-1}, \text{ where } x_i \neq x_j \text{ for } i \neq j\}.$ 
  - (a) Depict  $Q_n$  for n = 2, 3, 4.
  - (b) What is the cardinality of  $E_n$ , as a function of n?
  - (c) Is there anything notable about the degrees of vertices in  $Q_n$ ?
  - (d) Provide an alternate, equivalent (perhaps simpler) definition of  $Q_n$ .

7. Do the graphs  $Q_2$ ,  $Q_3$ , and  $Q_4$  of Question 6 have Eulerian paths? For each graph, exhibit an Eulerian path or explain why no such path exists.

Recall that an Eulerian path in a digraph is a directed path that traverses each edge exactly once. A digraph with such a path is called Eulerian.

8. Prove or disprove: The graphs  $Q_n$  of Question 6 are Eulerian for all n > 1.