This exercise is based on the paper\textsuperscript{1} on sorting posets with seven linear extensions.

1. List the members of your group below. Underline your name.

2. Provide methods to comparison-sort $n$ elements using the fewest comparisons, for $n = 4, 5, 6$. Justify your answers.

\textsuperscript{1}Satoshi Hanamura and Shigeki Iwata, “Posets with seven linear extensions sortable by three comparisons,” \textit{Information Processing Letters} 111/8 (2011).
3. Provide an illustrative example of a poset from a practical application. Prove that the relation you describe is a poset. How many linear extensions does this poset have? Depict its Hasse diagram.

4. Prove or disprove: If \( v \) is the unique maximum element of a poset \( P \) then \( e(P - v) = e(P) \), where \( P - v \) is the poset \( P \) with element \( v \) removed.
5. Depict Hasse diagrams of the posets $P_1[k]$ and $P_2[k]$, for $k = 1, 2, 3$, as defined by the paper.