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COS 497 Spring 2011 Class Exercise 1 8 questions; 4 pgs. Due 2011-01-11 12:15 p.m.
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1. List the members of your group below. Underline your name.
2. Describe your project in approximately 100 words and then have your partner critique your work. Exchange worksheets so that a description and its critique appear on the same one.
3. Critique from your partner:
4. Describe the current status of your project work. Be sure to include details of what is done and what remains to be done. Do you have a working demo? What does it include? Do you have a partial project report? What does the report include?
5. For an integer $n>1$, let $V_{n}$ be the set of ( $n-2$ )-character strings $\left\{x_{1} x_{2} \ldots x_{n-2} \mid x_{i} \in\right.$ $\{1,2, \ldots, n\}$ and $x_{i} \neq x_{j}$ unless $\left.i=j\right\}$.
(a) List $V_{n}$ for $n=2,3,4$.
(b) What is the cardinality of $V_{n}$, as a function of $n$ ?
(c) Provide an alternate, equivalent (perhaps simpler) definition of $V_{n}$.
6. For an integer $n>1$, define a digraph $Q_{n}=\left(V_{n}, E_{n}\right)$ where the set of vertices $V_{n}$ defined in Question 5 and the set of edges $E_{n}=\left\{(u, v) \mid u, v \in V_{n}\right.$ with $u=$ $x_{1} x_{2} x_{3} \cdots x_{n-2}, v=x_{2} x_{3} \cdots x_{n-2} x_{n-1}$, where $x_{i} \neq x_{j}$ for $\left.i \neq j\right\}$.
(a) Depict $Q_{n}$ for $n=2,3,4$.
(b) What is the cardinality of $E_{n}$, as a function of $n$ ?
(c) Is there anything notable about the degrees of vertices in $Q_{n}$ ?
(d) Provide an alternate, equivalent (perhaps simpler) definition of $Q_{n}$.
7. Do the graphs $Q_{2}, Q_{3}$, and $Q_{4}$ of Question 6 have Eulerian paths? For each graph, exhibit an Eulerian path or explain why no such path exists.

Recall that an Eulerian path in a digraph is a directed path that traverses each edge exactly once. A digraph with such a path is called Eulerian.
8. Prove or disprove: The graphs $Q_{n}$ of Question 6 are Eulerian for all $n>1$.

