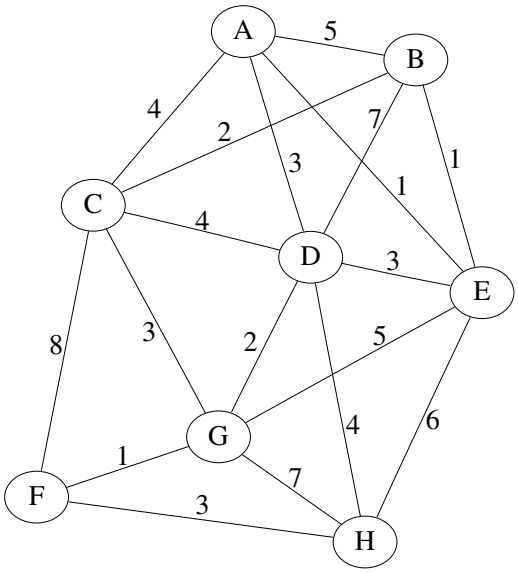
This exercise focuses on a detailed example of the use of two data structures, viz. the pairing heap and the union-find structure, to solve the minimum spanning tree problem using Kruskal's algorithm.

1. List the members of your group below. Underline your name.

2. An edge (u, v) is *named* uv if $u < v$ and vu otherwise. List all edges of the following graph in *lexicographic order of edge names*, noting the *weight* of each edge next to its name.



3. Depict the pairing min-heap obtained by inserting the (edge, weight) pairs of Question 2 in lexicographic order into an initially empty heap (using weights as keys). For this and the following questions, use the leftmost-child/right-sibling representation explicitly in your depiction.

4. Trace the action of two *deleteMin* operations applied to the heap of Question 3. Depict intermediate states and pairing-based reorganizations clearly.

5. Define a relation R on the vertices of the graph of Question 2 as follows: uRv (i.e., $(u, v) \in R$) iff the vertices u and v have the same degree. Prove or disprove: R is an equivalence relation.

6. Depict a *union-find data structure* with *path compression and union-by-rank* for the relation of Question 5. Use the notational and graphical conventions of Figure 24.18 in the textbook and build the structure by inserting vertices appropriately in sorted order by names.

7. Starting with the structure of Question6, depict the result of a *union* operation applied to merge the equivalence classes of the two lowest-named vertices that are not in the same class.

8. Starting with the final structure of Question7, depict the result of a *find* operation applied to locate the deepest vertex, breaking ties by choosing the lowest-named one.

9. Trace the operation of Kruskal's minimum spanning tree algorithm on the graph of Question 2. Indicate each edge that is examined and whether it is accepted or rejected. Depict, after each edge acceptance the states of:

   (a) the pairing heap used to organize unexamined edges,

   (b) the forest of the union-find data structure,

   (c) the forest of accepted edges forming the partial minimum spanning tree.

   Use the guidelines from earlier questions for the data structures.

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