Name: ________________________________

○ COS 480 students should answer non-⋆ questions; optional ⋆ questions are for extra credit.
○ COS 580 students should answer all questions, including ⋆ questions.

1. (1 pt.) Write your name in the space provided above.

2. (9 pts.) List all keys of the relation \( R(A, B, C, D, E, F) \) given the functional dependencies:

\[
AB \rightarrow C \quad (1) \\
C \rightarrow B \quad (2) \\
ADE \rightarrow C \quad (3) \\
EF \rightarrow AD \quad (4) \\
CE \rightarrow F \quad (5)
\]

Justify your answer by explaining why the listed attribute-sets are keys and why there are no other keys.
3. (20 pts.) Decompose the relation $R$ of Question 2 as needed to yield a BCNF schema. Clearly show all intermediate steps, including details such as the dependency used to decompose a relation, the resulting relations, their projected dependencies, and their keys.
[additional space for answering the earlier question]
4. (10 pts.) Use the chase test to determine whether the decomposition of the relation $R$ from the schema of Question 2 into $R_1(A, B, C)$, $R_2(A, C, E)$, $R_3(B, D, E)$, and $R_4(A, B, F)$ is lossless. Clearly indicate the operations on the tableau and the dependency used for each operation.
5. (10 pts.) Use the chase test to prove or disprove each of the following for a relation \( S(A, B, C, D, E, F) \). Clearly indicate the operations on the tableau and the dependency used for each operation.

(a) If \( AB \rightarrow CD \) then \( AB \rightarrow EF \).
(b) If \( AB \rightarrow C \) and \( AC \rightarrow B \) then \( BC \rightarrow A \).
[additional space for answering the earlier question]
6. (10 pts.) Provide algebra and Datalog equivalents of the following SQL query on a database with relations $T(A, B, C)$ and $U(B, D, E, F)$, or prove that no equivalents exist. (Assume that the types of all attributes used in comparison predicates are compatible.)

```
from T T1, T T2, U
where T1.B = U.B and T2.B = some (select E from U where F < T1.B)
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7. ★ (20 pts.) Prove or disprove: There exists a relational algebra expression that uses no operators other than \( \sigma, \pi, \times, \cup, -, \rho \) that is equivalent to the SQL query

\[
\text{select A, B from R}
\text{where B = (select min(B) from R where C > 20)}
\]

To prove, you must provide an algebra expression and prove its equivalence to the SQL query. To disprove, you must clearly articulate why no such expression is possible.