1. (1 pt.)

- **Read all material carefully.**
- You may refer to your books, papers, and notes during this test.
- No computer or network access of any kind is allowed (or needed).
- Write, and draw, carefully. Ambiguous or cryptic answers receive zero credit.
- Use textbook and classroom conventions for notation, algorithmic options, etc.
- Ask for clarifications on the above if needed.

Write your name in the space provided above.

2. (14 pts.) Depict **all** red-black trees that contain exactly the five keys 1, 2, 3, 4, and 5. As in class, use circles for red nodes and boxes for black nodes. Briefly **explain** why the trees you depict are the only possibilities. If there are more than 10 trees that qualify, depict any 10 of your choice.
[additional space for answering the earlier question]
3. (15 pts.) For each tree of Question 2:

(a) Redraw the tree below ignoring node colors. Depict each node as a circle enclosing its key.

(b) Write the height of each node to the right of its circle.

(c) If the resulting tree is a valid AVL-tree, write AVL next to its root. Otherwise, write not AVL next to its root and mark AVL-unbalanced nodes with an asterisk *. 
4. (15 pts.) [This question is similar to Question 3, but uses a mapping to AA trees instead of to AVL trees.] For each tree of Question 2:

(a) Redraw the tree below with each red node drawn as a horizontal child of its parent (cf. AA-trees); then ignore colors. Black nodes remain vertical children of their parents as before. Depict each node as a circle enclosing its key and be sure to depict horizontal children clearly.

(b) If the resulting tree is a valid AA-tree then:

   i. Write AA next to its root.
   ii. Write the AA-tree level of each node to the right of its circle.

else:

   i. Write not AA next to its root.
   ii. Mark with an asterisk * all feature that violate AA-tree properties.
5. (15 pts.) For each valid AA-tree of Question 4:

(a) Present a list of keys that produces the tree when inserted in sequence into an initially empty tree.

(b) Depict the action of the above insertions. Mark any skew and split operations used. Depicting the state of the tree after each insertion.
[additional space for answering the earlier question]