1. (1 pt.)
   - Read all material carefully.
   - You may refer to your books, papers, and notes during this test.
   - No computer or network access of any kind is allowed (or needed).
   - Write, and draw, carefully. Ambiguous or cryptic answers receive zero credit.
   - Use the conventions used in class and the textbook for all material.

Write your name in the space provided above.

2. (9 pts.) A proper $k$-coloring of a graph is an assignment of colors $1, 2, \ldots, k$ to the vertices of the graph such that no two neighboring vertices have the same color. A graph is said to be $k$-colorable if it admits a proper $k$-coloring.

Write a Datalog query that tests the 2-colorability of a connected graph that is represented by its edges in a relation $Edges(s, d)$. Briefly explain why your query is correct.

Hint: A graph is 2-colorable iff it does not contain a cycle of odd length.
3. (10 pts.) Provide recursive-SQL query that is equivalent to the query of Question 2. Briefly explain why your query is correct.
4. (20 pts.) Consider a relation \( R(A, B, C, D, E, F) \) with the following basis of dependencies (note carefully: FDs v. MVDs):

\[
\begin{align*}
A & \rightarrow BC \\
CD & \rightarrow A \\
D & \rightarrow E \\
F & \rightarrow B \\
AC & \rightarrow E
\end{align*}
\]

(a) Provide an instance of \( R \) that violates the dependency \( CD \rightarrow A \) without violating any of the other dependencies.

(b) List all keys of \( R \).

(c) Explain your answer, noting why the keys you list are valid and also why there are no other keys.

(d) How many superkeys does \( R \) have? Explain your answer. (You need not list all superkeys.)
5. (15 pts.) Decompose the schema of Question 4 to 4NF. Show all intermediate steps and details, as in class exercises (keys, projected dependencies, decomposed relations, etc.).

\[
\begin{align*}
A & \rightarrow BC \\
CD & \rightarrow A \\
D & \rightarrow E \\
F & \rightarrow B \\
AC & \rightarrow E
\end{align*}
\]
[additional space for answering the earlier question]
6. (5 pts.) Given a database with table \( R(\text{A, B, C, D, E, F}) \), with all attributes of type integer, provide the simplest SQL statements to declare the following constraints:

(a) Attributes A and B must not be null.
(b) The sum of C and D must be no greater than E.