Name: ____

1. (1 pt.)

   - **Read all material carefully.**
   - You may refer to your books, papers, and notes during this test.
   - No computer or network access of any kind is allowed (or needed).
   - Write, and draw, carefully. Ambiguous or cryptic answers receive zero credit.
   - Use textbook and classroom conventions for notation, algorithmic options, etc.
   - Ask for clarifications on the above if needed.

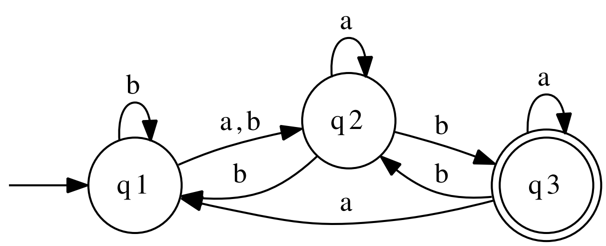
   Write your name in the space provided above.

2. (14 pts.) Use the method described in the proof of Lemma 1.60 in the textbook to convert the following regular expression to an NFA. *Depict intermediate steps and ensure you follow that method.*

$$(a \cup ba \cup cc*)a$$

[additional space for answering the earlier question]

3. (15 pts.) Generate a regular expression that is equivalent to the following finite-state automaton. *Show enough intermediate results and include brief explanations* to make it clear that the method described in the textbook is being followed.



[additional space for answering the earlier question]

4. (15 pts.) Convert the following grammar to Chomsky normal form. Upper-case letters denote variables and lower-case letters denote terminals. *Show enough intermediate results and include brief explanations* to make it clear that the method described in the textbook is being followed.

$$
\begin{aligned}
S &\rightarrow AaB \mid BbA \mid aSa \mid bSSb \\
A &\rightarrow ab \mid aaA \mid \epsilon \\
B &\rightarrow b \mid Sb \mid \epsilon
\end{aligned}
$$

[additional space for answering the earlier question]

5. (15 pts.) Using the tabular representation used in class, depict the operation of the CYK algorithm on the input string `aabaabaaaa` and the final (Chomsky normal form) grammar of Question 4.

6. (15 pts.) Prove or disprove: The language defined by the grammar of Question 4 is regular.

7. (15 pts.) Provide a game of *generalized geography* (as defined in Section 8.3 of the textbook) in which the first player can always win. Use a directed graph that has *at least 10 vertices*, all but one of which should have an *out-degree of at least 3*. *Explain* briefly why your answer is correct.

8. (20 pts.)

   (a) Reduce the following SAT instance to a SUBSET-SUM using the textbook's method.

   (b) Depict corresponding solutions to the instances, or explain why none exist.

$$(x \vee y \vee \bar{z}) \wedge (\bar{x} \vee \bar{y} \vee z) \wedge (x \vee \bar{y} \vee \bar{z}) \wedge (\bar{x} \vee y \vee \bar{z}) \wedge (x \vee \bar{y} \vee z)$$

[additional space for answering the earlier question]