**Today:** shell sort; polyphase merging. §§8.1–8.4; Reynolds's paper.[1]
**Next class:** Skew and pairing heaps. §§23.*. Reminder: Read *before and after* class.

1. List the members of your group below. Underline your name.

2. A *subsequence* of sequence $S$ is any sequence that can be obtained from $S$ by delet-ing zero or more of its elements. For example, $(1, 4, 9, 2)$ is a subsequence of $S_1 = (3, 1, 4, 5, 9, 2)$, but $(1, 9, 4)$ is not. A subsequence $S'$ of $S$ is called a *k-subsequence* if each pair of adjacent elements in $S'$ has $k-1$ intermediate elements in $S$. For example, $(1, 5, 2)$ is a 2-subsequence of $S_1$, and $(3, 5)$ is a 3-subsequence of $S_1$, but $(1, 5, 9)$ is not a $k$-subsequence of $S_1$ for any value of $k$ (although it is a subsequence of $S_1$). A $k$-subsequence with $n$ elements is called *maximal* if there is no $k$-subsequence with $n + 1$ elements. List all *maximal 5-subsequences* and *maximal 7-subsequences* of the following sequence:

$$50 \quad 40 \quad 60 \quad 70 \quad 65 \quad 75 \quad 62 \quad 63 \quad 41 \quad 42 \quad 51 \quad 52 \quad 53 \quad 54$$

3. We say a sequence is *k-sorted* if all of its k-subsequences are sorted. For each of the following, provide an example of a sequence with the indicated properties, or explain why no such sequence exists.

   (a) 7-sorted but not 5-sorted.
   (b) 5-sorted but not 7-sorted.
   (c) 6-sorted but not 3-sorted.
   (d) 3-sorted but not 6-sorted.

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[1]Samuel W. Reynolds, "A Generalized Polyphase Merge Algorithm," *Communications of the ACM* 4/8 (1961).

4. Sort the following array in ascending order using *shellsort with increment sequence* $(1, 5, 7)$.[2] Depict the state of the array after each $k$-sort, for $k = 1, 5, 7$ and highlight the moved elements at each stage.

$$50 \quad 40 \quad 60 \quad 70 \quad 65 \quad 75 \quad 62 \quad 63 \quad 41 \quad 42 \quad 51 \quad 52 \quad 53 \quad 54$$

5. Refer to Reynolds's paper[3] on generalized polyphase merging. List the first 20 k-generalized Fibonacci numbers for $k = 2, 3, 4, 5$.

6. (informal homework) Using ideas from Reynolds's paper on generalized polyphase merging, describe how to determine the initial distribution of sorted runs on tapes for a $k$-way polyphase merge sort. Provide illustrative examples.

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[2], §8.4.
[3]Reynolds, *op. cit.*