COS 226 Fall 2016 Class Exercise $9 \quad 5$ questions; 6 pgs. 2016-11-01

Today: splay trees; $\S \S 22 . *$.
Next class: pairing heap; $\S \S 23 .\{1,2\}$.

1. Write your group members' names below. Underline your name.
2. In order to speed up access to frequently or recently accessed elements, the rotate-to-root method of maintaining a binary search tree (that is not explicitly balanced) requires each element that is accessed to be rotated with its parent, if any, until the accessed element is the root of the tree. Thus, if the same element is accessed again immediately it requires only one tree node to be visited. If a few other operations intervene, the previously accessed elements are still likely to be close to the root.
Depict the transformations to the following binary search tree resulting from the above method applied in response to the access pattern $1,2,3,4,5$. Depict all rotations and highlight the tree after all rotations for each insertion have completed.

[additional space for answering the earlier question]
3. What is the total number of rotations in the transformation of Question 2?

Generalize the example of that question to a sequence of accesses $1,2, \ldots, n$ applied to an initial left-only tree with keys $n, n-1, \ldots, 1$ and express the number of rotations as a function of $n$.
4. Repeat Question 2 using the standard bottom-up splay tree operations of zig, zig-zag, and zig-zig instead of the rotate-to-root method of that question. Indicate the type of operation used in each step of the transformation.
[additional space for answering the earlier question]
5. Depict the transformations to the following top-down splay tree in response to the access pattern $1,2,3,4,5$. Depict all splay operations clearly, including the left and right trees, and highlight the tree after all operations for each insertion have completed.

[additional space for answering the earlier question]

