

Name: _____

1. (1 pt.)

- **Read all material carefully.**
- Budget your time: 60 minutes, 60 pts \Rightarrow 1 min./pt. avg.
- You may refer to your books, papers, and notes during this test.
- No computer or network access of any kind is allowed (or needed).
- Write, and draw, carefully. Ambiguous or cryptic answers receive zero credit.
- Use the conventions used in class and the textbook for notation, algorithmic options, etc.
- There is one extra-credit question at the end. It is marked with a ★ and is harder (and graded more strictly) than the rest. It is inadvisable to spend time on it before finishing the rest.
- Some questions are on *search trees* and others are on *heaps*; be sure you're using the correct one for each question.

Write your name in the space provided above.

[Proceed to next page when indicated.]

2. (14 pts.) Depict the sequence of *bottom-up splay tree* states resulting from the insertion of the following keys, in the order presented, into an initially empty tree.

52, 77, 45, 85, 48

You must depict intermediate tree states, including the state after each insertion, clearly marking and identifying each rotation (zig, zig-zig, zig-zag). [Hint: For each insertion, perform a standard binary-search-tree insertion, followed by a sequence of rotations to bring the new item to the root.]

[additional space for answering the earlier question]

3. (15 pts.) Repeat Question 2 for a *top-down splay tree*. In addition to the directions of that question, be sure to *clearly mark the left, middle, and right trees* using the three-column format we used in class (and from the textbook).

52, 77, 45, 85, 48

[Hint: The hint of Q. 2 does not apply here. Instead, the rotations must be performed along the way, top-down, as we proceed to the location of the item being inserted.]

[additional space for answering the earlier question]

4. (15 pts.)

- (a) Depict the sequence of *skew heap* states resulting from the insertion of the following keys, in the order presented, into an initially empty heap.

52, 77, 45, 85, 48, 31, 88, 99

- (b) Depict the result of two *deleteMin* operations on the final heap above.

Depict the state of the heap after each insert and deleteMin operation (at least).

[additional space for answering the earlier question]

5. (15 pts.) Repeat Question 4 (both parts) for a *pairing heap*.

Use the *abstract representation* (only) of pairing heaps in your figures.

However, depict the final tree in both abstract and actual (concrete) representations.

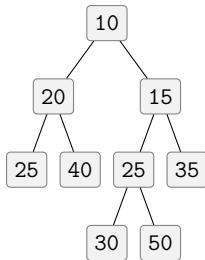
52, 77, 45, 85, 48, 31, 88, 99

[additional space for answering the earlier question]

6. (10 ★ pts.) [Reminder: Extra credit. Harder. Graded more strictly.]

We define a *generating sequence* $S = k_1, k_2, \dots, k_n$ of a skew heap h to be sequence of keys that produces h when the keys are inserted into an initially empty heap in sequence order.

- (a) How many generating sequences does the skew-heap depicted below have? *Justify your answer.* (There is no credit otherwise.)
- (b) Depict all the above generating sequences (or any 10, if there are more than 10 of them).
- (c) Trace the insertions for the lexicographically smallest of the generating sequences.



[additional space for answering the earlier question]