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COS 350 Spring 2017 Midterm Exam 1 60 pts;; 60 minutes; 7 questions; 9 pages. 2017-02-23 12:30 p.m.
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Name: $\qquad$

1. (1 pt.)

- Read all material carefully.
- If in doubt whether something is allowed, ask, don't assume.
- You may refer to your books, papers, and notes during this test.
- E-books may be used subject to the restrictions noted in class.
- No computer or network access of any kind is allowed (or needed).
- Write, and draw, carefully. Ambiguous or cryptic answers receive zero credit.
- Use class and textbook conventions for notation, algorithmic options, etc.
- There is an extra-credit question (marked with $\star$ ). It is harder than the rest.
- Budget your time: roughly one minute per point.

Write your name in the space provided above.
2. ( 9 pts.) For the following mapping of rod lengths to prices, how many recursive invocations of Cut-Rod does the recursive top-down cut-rod algorithm make? Provide an exact numerical answer along with an explanation. [Hint: You do not need to solve the cut-rod instance.]

| length: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| price: | 4 | 7 | 9 | 14 | 18 | 22 | 30 | 30 | 28 | 38 | 40 | 44 |

3. (10 pts.) Solve the following recurrences. Clearly state the methods you use for your solutions and outline their key steps. (Show your work.)
(a) $T(n)=2 T(n / 2)+3 n+1$
(b) $S(n)=7 S(n / 2)+8 n \sqrt{n}$
[additional space for answering the earlier question]
4. (10 pts.) Trace the operation of the LCS-LENGTH algorithm on the following sequences.

$$
\begin{array}{lllllll}
A & C & B & A & A & B & A \\
C & B & A & C & A & A & B
\end{array}
$$

Depict the state of the $b$ and $c$ arrays (1) after four iterations of the outer nested loop and (2) at the end of the algorithm.
[additional space for answering the earlier question]
5. (10 pts.) Trace the operation of the Print-LCS algorithm on the result of Question 4. Provide the arguments for each of recursive call of Print-LCS.
6. (10 pts.) Depict tables similar to those in Figure 15.5 of the textbook for Matrix-Chain-Order on the following input:

| matrix: | $A_{1}$ | $A_{2}$ | $A_{3}$ | $A_{4}$ | $A_{5}$ |
| ---: | :---: | :---: | :---: | :---: | :---: |
| dimension: | $10 \times 30$ | $30 \times 50$ | $50 \times 100$ | $100 \times 40$ | $40 \times 30$ |

7. (10 pts.) Depict the first three levels of the recursion tree that outlines the recursive calls made by the Find-Maximum-Subarray algorithm when invoked on the following array, with low and high equal to 1 and 10 , respectively.
The nodes of the tree should be labeled with the function invoked: Find-MaximumSubarray ( $M$ ) or Find-Max-Crossing-Subarray ( $X$ ).

The edges should connect each function's node (child) to the node of its invoker (parent).

| i: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| A [i]: | 88 | -1 | -11 | -23 | 43 | -6 | 8 | -19 | -58 | 50 |
|  |  |  |  |  |  |  |  |  |  |  |

[additional space for answering the earlier question]

