

Name: \_\_\_\_\_

1. (1 pt.)

- **Read all material carefully.**
- Budget your time: 60 minutes, 60 pts  $\Rightarrow$  1 min./pt. avg.
- You may refer to your books, papers, and notes during this test.
- No computer or network access of any kind is allowed (or needed).
- Write, and draw, carefully. Ambiguous or cryptic answers receive zero credit.
- Use the conventions used in class and the textbook for notation, algorithmic options, etc.
- There is one extra-credit question at the end. It is marked with a ★ and is harder than the rest.

Write your name in the space provided above.

2. (9 pts.) Determine **both** the **exact** and the **asymptotic** number of times the statement tagged *count me* below is executed. Express your answers as functions (as concise and simple as possible) of  $n$  and justify them briefly.

```
1   int boo = 0;
2   for(int i = 0; i < n; i++) {
3       for(int j = i; j < n; j++) {
4           for(int k = j; k < n; k++) {
5               for(int z = 10; z > 0; z--) {
6                   boo = boo + (i * j) + (k * z); /* count me */
7               }
8           }
9       }
10  }
```

3. (10 pts.) Depict the sequence of *AVL* tree states resulting from the insertion of the following keys, in the order presented, into an initially empty tree.

50, 25, 32, 70, 90, 40, 80, 27, 45, 47, 48

You must depict intermediate tree states, including the state after each insertion, clearly marking and identifying each rotation.

[additional space for answering the earlier question]

4. (10 pts.) Repeat Question 3 for *bottom-up red-black* trees. Follow the graphical conventions used in class: round nodes for red and boxed nodes for black. You must depict intermediate tree states, including the state after each insertion, clearly marking and identifying each rotation and color change.

50, 25, 32, 70, 90, 40, 80, 27, 45, 47, 48

[additional space for answering the earlier question]

5. (10 pts.) Repeat Question 4 for *top-down* red-black trees,  
You must depict intermediate states as outlined in Question 4.

50, 25, 32, 70, 90, 40, 80, 27, 45, 47, 48

[additional space for answering the earlier question]

6. (10 pts.) Repeat Question 5 for *AA* trees,

You must depict intermediate tree states, including the state after each insertion, clearly marking and identifying each skew and split operation.

Ensure that horizontal and vertical links are drawn clearly and with arrows.

50, 25, 32, 70, 90, 40, 80, 27, 45, 47, 48



[additional space for answering the earlier question]

7. (10 pts.) Depict the result of deleting the following keys, in the order presented, from the final tree of Question 6.

45, 50, 32, 80

As before, depict the state of the tree after each deletion and clearly mark and identify each split and skew operation.

[additional space for answering the earlier question]

8. (10 ★ pts.) Depict **all** AA-trees that contain exactly the five keys: 1, 2, 3, 4, 5. Briefly **explain** why the trees you depict are the only possibilities. If there are more than 10 trees that qualify, depict any 10 of your choice and describe clearly how the remaining ones may be obtained.

[additional space for answering the earlier question]