Name: _____

1. (1 pt.)

   - **Read all material carefully.**
   - Budget your time: 55 minutes, 55 pts $\Rightarrow$ 1 min./pt. avg.
   - You may refer to your books, papers, and notes during this test.
   - Electronic books are allowed *only when used as described in class.*
   - No computer or network access of any kind is allowed (or needed).
   - Write, and draw, carefully. Ambiguous or cryptic answers receive zero credit.
   - Use the conventions used in class and the textbook for notation, algorithmic options, etc.
   - There is one extra-credit question at the end. It is marked with a ★ and is harder than the rest (and graded more strictly).

   Write your name in the space provided above.

Wait for the go signal before proceeding to next page.

2. (9 pts.) Depict the sequence of *binary search tree* states (without any balancing operations) resulting from the insertion of the following keys, in the order presented, into an initially empty tree. You must depict intermediate tree states, including the state after each insertion.

$$50, 30, 20, 25, 22, 24, 23$$

[additional space for answering the earlier question]

3. (10 pts.) Repeat Question 2 for *AVL* trees. You must depict intermediate tree states, including the state after each insertion, clearly marking and identifying each rotation.

$$50, 30, 20, 25, 22, 24, 23$$

[additional space for answering the earlier question]

4. (10 pts.) Repeat Question 3 for *bottom-up red-black* trees. Follow the graphical conventions used in class: round nodes for red and boxed nodes for black. You must depict intermediate tree states, including the state after each insertion, clearly marking and identifying each rotation and color change.

$$50, 30, 20, 25, 22, 24, 23$$

[additional space for answering the earlier question]

5. (10 pts.) Repeat Question 4 for *AA* trees,

You must depict intermediate tree states, including the state after each insertion, clearly marking and identifying each skew and split operation. Ensure that horizontal and vertical links are drawn clearly and with arrows. Use dashed lines to demarcate levels unless your drawing is precise enough to make the levels apparent.

$$50, 30, 20, 25, 22, 24, 23$$

[additional space for answering the earlier question]

[additional space for answering the earlier question]

6. (15 pts.) Determine **both** the **exact** and the **asymptotic** number of times the statement tagged *count me* below is executed.

Express your answers as functions (as concise and simple as possible) of n and justify them briefly.

You are permitted to use any results from the textbook that you find helpful.

```
1      int goo = 0;
2      for(int i = 0; i < n; i ++) {                    // see text for 'n'
3          for(int j = i; j < n; j++) {
4              for(int k = j; k < n; k++) {
5                  for(int z = 3; z > 0; z--) {
6                      goo = goo + (i * j) + (k * z); /* count me */
7                  }
8              }
9          }
10     }
```

[additional space for answering the earlier question]

7. (10 ⋆ pts.) Recall the triple-based representation of binary trees from class and the homework:

> Let us represent the empty binary tree by ⊥ and a nonempty binary tree with root label n, left subtree l, and right subtree r by the triple (n, l, r).

Using the above and the conventions of the homework, define functions s and d such that $s(t)$ performs an *AVL single rotation* on the subtree rooted at t, and similarly $d(t)$ performs an *AVL double rotation.*