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1. (1 pt.)
   - **Read all material carefully.**
   - *If in doubt whether something is allowed, ask, don't assume.*
   - You may refer to your books, papers, and notes during this test.
   - Write, and draw, carefully. Ambiguous or cryptic answers receive zero credit.
   - Use class and textbook conventions for notation, algorithmic options, etc.
   - For the duration of the exam, the only communication (live or network) should be with the instructor for clarifications, etc.
   - At the end of the exam, scan your work to a PDF file named using the following template and upload it in the usual way:
     `cos451-mt02-`*`lastname`*`-`*`firstname`*`-pqrs.pdf`
     (replacing *lastname* and *firstname* with yours and *pqrs* with an arbitrary 4-digit number).

   Write your name in the space provided above.

WAIT UNTIL INSTRUCTED TO CONTINUE TO REMAINING QUESTIONS.

Do not write in the following table.

Q	Full	Score
1	1	
2	19	
3	20	
total	40	

2. (19 pts.) Convert the following grammar to Chomsky normal form. Upper-case letters represent variables and lower-case letters denote terminals. *Show enough intermediate results and include brief explanations* to make it clear that the method described in the textbook is being followed.

$$A \rightarrow aa \mid ABA$$
$$B \rightarrow b \mid bAbA \mid \varepsilon$$

~~$r_1 : A \rightarrow aa$~~
~~$r_2 : A \rightarrow ABA$~~
$r_3 : B \rightarrow b$
~~$r_4 : B \rightarrow bAbA$~~
~~$r_5 : B \rightarrow \varepsilon$~~

Step 1: New start symbol S_0 & rule:

~~$r_0 : S \rightarrow A$~~

Step 2: ε-rule elim. (r_5)

delete r_5, add: ~~nothing (= no rule~~

$r_{2.1} : A \rightarrow AA$

Step 3: unit rules elim. (only r_0)

delete r_0; add:

~~$r_{0.1} : S \rightarrow aa$~~
~~$r_{0.2} : S \rightarrow ABA$~~

Step 4: Convert rule RHSs to proper form

(rules $r_{0.1}$, $r_{0.2}$, r_1, r_2, r_4)

($r_{2.1}$ and r_3 are OK)

[additional space for answering the earlier question]

delete $r_{0.0.1}$, add:

$r_{0.0.1.0.1}$: $S \to \cancel{a} X_1 X_1$

$r_{0.0.1.1.2}$: $X_1 \to a$

delete $r_{0.2}$, add

$S \to A X_2$ $\quad r_{0.2.1}$

$X_2 \to B \cancel{A}$ $\quad r_{0.2.2}$

delete r_1, add:

$r_{1.0.1}$: $A \to \cancel{a} X_3$

$r_{1.1.2}$: $X_3 \to a$

delete r_2, add: (could simplify)

$r_{2.0.2}$: $A \to A X_4$

$r_{2.1.3}$: $X_4 \to BA$

delete r_4, add:

$r_{4.0.1}$: $B \to \cancel{b} X_5 X_6$

$r_{4.1.2}$: $X_5 \to b$

$r_{4.1.3}$: $X_6 \to A X_7$

$r_{4.1.4}$: $X_7 \to X_5 A$

Final CNF grammar: (rules r_1 -- r_{12})

r_1: $S \to \cancel{a} X_1 X_1$

r_2: $X_1 \to a$

r_3: $S \to A X_2$ (simplify)

r_4: $X_2 \to BA$

r_5: $A \to \cancel{a} X_1 X_1$

~~$X_3 \to a$~~

r_6: $A \to A X_4$

r_7: $X_4 \to BA$

r_8: $B \to b$

~~$B \to \cancel{b} X_6$~~

~~$X_6 \to A X_7$~~

~~$X_7 \to BA$~~

r_9: $B \to X_5 X_6$

r_{10}: $X_5 \to b$

r_{11}: $X_6 \to A X_7$

r_{12}: $X_7 \to X_5 A$

3. (20 pts.) Let G_2 be the grammar of Question 2. For each of the following strings, indicate whether the string belongs to $L(G_2)$. If so, provide a leftmost derivation of that string. Otherwise, prove as precisely as possible that the string does not belong to $L(G_2)$.

(a) aabaabaaaa $= S_1$

(b) aabbaba $= S_2$

$r_1 : A \to aa$

$r_2 : A \to ABA$

$r_3 : B \to b$

$r_4 : B \to bAbA$

$r_5 : B \to \varepsilon$

$\overset{i}{\Longrightarrow}$ = using rule r_i

(a) $\underline{A} \overset{2}{\Longrightarrow} \underline{A}BA \overset{1}{\Longrightarrow} aa\underline{B}A \overset{4}{\Longrightarrow} aab\underline{A}bAA$

$\overset{1}{\Longrightarrow} aabaab\underline{A}A \overset{1}{\Longrightarrow} aabaabaa\underline{A}$

$\overset{1}{\Longrightarrow} aabaabaaaa$

So $S_1 \in L(G_2)$

(b) $S_2 \notin L(G_2)$ lone

S_2 contains an "a" (i.e., an "a" that is not next to another "a"). (Actually 2 such instances)

However, there is only one rule in G_2 that produces (directly) "a"s (r_1) and that always results in pairs of adjacent "a"s. Therefore no string $w \in L(G_2)$ can have lone "a"s $\Rightarrow S_2 \in L(G_2)$ ($\because$ has lone "a")