

**Today** Preliminaries and more. § 0.\* (thoroughly); § 6.1 (the best you can).

**Next class** Finite-state automata (FSAs). §§ 1.1, 1.2.

1. Write your name below.

2. Let  $A = \{1, 2, 4, 8, 16, \dots, 1024\}$  and  $B = \{n \in \mathbb{Z} \mid 0 < n \leq 100 \wedge \sqrt{n} \in \mathbb{Z}\}$ .

(a) Provide a compact implicit definition of  $A$ .

(b) Enumerate the elements of  $B$ .

(c) Enumerate each of the following. You may abbreviate if the result is clear and unambiguous.

i.  $A \cup B$

ii.  $A \cap B$

iii.  $A \setminus B$

iv.  $A \times B$

v.  $\mathcal{P}(B)$

3. With all variables ranging over the set  $\mathbb{Z}$ , for each of the following logical sentences, (1) provide a brief but precise English equivalent, (2) provide a prenex normal form equivalent, and (3) either prove or disprove it.

(a)  $\forall y \exists x [\nexists w [w = x^2] \wedge \exists z [x < y < z]]$

(b)  $\exists x \forall y [\nexists w [w = x^2] \wedge \exists z [x < y < z]]$

4. Use an illustrative example to explain the difference between *lexicographic* and *shortlex* ordering of strings.

5. Depict the graphs used in the proof of Theorem 0.22 for  $n = 4, 6, 8$ .

6. Prove or disprove:  $(x + y)/2 \geq \sqrt{xy}$  for all nonnegative reals  $x$  and  $y$ .