This assignment is related to the classroom discussion on bisecting trees, in turn related to Problem B-3 in Appendix B of the textbook.[1] Completed assignments should be submitted in hardcopy form (using as many sheets of Letter sized paper as needed) in class on the due date. *All work should be very well organized and presented; otherwise it will earn a score of 0 with no detailed grading or feedback.*

Items tagged with (554) are optional for COS 454 but required for COS 554.

1. Consider the statement of Problem B-3.a from the textbook:

   > Show that the vertices of any n-vertex binary tree can be partitioned into two sets A and B, such that $|A| \leq 3n/4$ and $|B| \leq 3n/4$, by removing a single edge.

   (a) (25 pts.) Make the smallest possible change to the above statement (including possibly no change) that makes it true. Explain in detail why the change is necessary and also why it is the smallest possible one.

   (b) (25 pts.) Prove the statement, possibly modified as above.

2. Refer to the textbook's pseudocode for INSERTION-SORT.

   (a) (25 pts.) Modify the pseudocode (as little as possible) to operate from the other end of the array. That is, the sorted subarray should begin at the highest array index and grow towards lower indices (right to left instead of left to right). Present your pseudocode using the same format and conventions as the textbook. Briefly explain why it is correct (informally).

   (b) (25 pts.) Using the textbook's partial proof of its correctness as a guide, prove the correctness of the above (modified) pseudocode *fully* (including correctness of the inner loop, in particular).

3. (554; 25 pts.) Recall class discussion related to Problem B-3.b from the textbook. Prove or disprove the following:

   > For every integer k there is a binary tree T with $|T| \geq k$ such that removing a single edge from T results in trees T_1 and T_2 with $|T_1| \leq |T_2|$ and $|T_2| \geq 3|T|/4$.

[1] Thomas H. Cormen et al., *Introduction to Algorithms*, 4th edition (MIT Press, 2022).