# Beacon Placement for Indoor Localization using Bluetooth

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*Abstract*— **We describe a method for determining the location of a mobile device, such as a handheld computer or mobile phone, in an indoor environment using Bluetooth beacons. Since it uses inexpensive commodity devices, this method is inexpensive to deploy. The limited range of Bluetooth reception is used to advantage. Another important advantage of this method is that it allows the mobile device to determine its location while remaining anonymous, unidentified to the beacons or other nearby devices. In such a deployment, an important design task is the placement of beacons. Signal propagation in indoor environments is complex, affected by factors such as floor-plans and duct-work, varying transmission and reflection properties of building materials and furniture, and interference from other devices. Therefore, the area from which a beacon is visible is very irregular and not well approximated by simple models such as ellipsoids. Our solution permits complex reception characteristics to be accurately modeled and provides a simple method for choosing beacon locations.**

## I. Introduction

Localization refers to the task of determining the location of a traveler in a specified coordinate system, which is subject to topological constraints, using a mobile device carried by the traveler. Perhaps the most common example is terrestrial localization in the WGS84 reference frame using a mobile GPS receiver and the infrastructure of GPS satellites [1]. Recent years have witnessed a rapid commoditization of GPS hardware and related products, as well as other similar technologies, so that outdoor localization is now inexpensive and accessible. *Indoor localization* refers to the task of localizing a traveler, using a suitable mobile device, in typical indoor environments, such as office buildings, airports, and railway stations. Although there have been notable advances in this area as well, the progress in indoor localization has lagged that in outdoor localization. For example, while it is possible to purchase a device for less than 100 USD that provides not only location information over several countries but also features such as mapping and navigation, no such option exists for localization in major airports, railway stations, and large office buildings.

In some aspects, indoor localization is simpler than outdoor localization. For instance, the geographical area covered is much smaller, and the expected speeds of travel are much lower. However, other aspects make indoor localization much more challenging. Chief among these is the unsuitability of GPS and related technologies. In addition to problems receiving reliable signals from satellites, or other outdoor beacons, in complex indoor environments, the requirements of indoor localization are also more stringent. For example, a vertical positioning uncertainty of several meters does not pose much of a problem for outdoor GPS applications such as route guidance for cars. However, inside a building, that uncertainty translates into uncertainty in the floor of the building. More generally, topological constraints in indoor environments are much more complex than those in outdoor environments. For these and other reasons, prior work on indoor localization has looked to several alternative methods, such as the use of visual markers that are detected by mobile-phone cameras, ultrasonic signals, RFID, and 802.11. Each of these methods has its strengths and weaknesses and, in this paper, our focus is on using Bluetooth signals for indoor localization.

The Bluetooth standard was devised for short-range (few meters) communication and therefore most Bluetooth devices have very limited range. The short range, compared to alternatives such as 802.11, has some disadvantages: For instance, a much larger number of devices must be deployed in order to provide adequate coverage. However, the short range is also a significant advantage. One of the problems identified by prior work on 802.11-based localization is that it is extremely difficult to accurately judge the distance from a beacon, using signal strength or other properties, due to complex signal propagation artifacts. With the short range of Bluetooth devices, this problem is significantly mitigated. In effect, if a mobile device detects a Bluetooth beacon then it is very likely that the beacon is only a few meters away. In contrast, an 802.11 beacon may be detected even if it is two buildings away. Our work takes advantage of this feature to yield an indoor localization scheme that is inexpensive and easy to deploy. Another benefit of Bluetooth is that the transceivers are extremely inexpensive, costing less than 3 USD in bulk.

Our method is based on deploying a large number of very inexpensive Bluetooth-based beacons (costing roughly 5 USD each). Localization is performed using a cell-based method that determines the region of intersection of visible beacon ranges. We describe this process in Section II. An important question that arises in this situation is where the beacons should be placed in order to achieve the most accurate localization for a given number of beacons. We describe this problem in Section III and model it formally as the problem of finding a maximum-resolution sub-hypergraph. Section IV addresses the solution of this problem. We

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describe some implementation issues in Section V, including a description of our beacons. We discuss related work in Section VI and conclude in Section VII.

## II. Localization using Bluetooth Beacons

Localization using beacons may be achieved using a variety of techniques, such as triangulation, trilateration, multilateration, and cell-based methods. Triangulation requires the measurement of angles between the line connecting two beacons and the line of sight from each of the beacons to the traveler. It is therefore not suitable for Bluetooth and other radio frequency technologies, but may be successfully applied for a system of visual beacons. The related rho-theta method, based on measuring the distance and angle to a single beacon, suffers from a similar drawback for Bluetooth. We note that sometimes the term triangulation is used as a synonym for trilateration, described next, but the two are quite different.

Trilateration requires the computation of the distance of the traveler from each of three beacons. The traveler's position is then uniquely determined as the point of intersection of three circles, each centered at one of the beacons. The radius of each of these circles is the computed distance of the traveler from that beacon. Computing the distance between Bluetooth devices is problematic. Prior work has explored the use of signal strength as a proxy for distance, but the results are not encouraging [2], [3], [4], [5], [6], [7]. In general, the correlation between distance and signal strength is not sufficiently high because of a variety of radio artifacts, as well implementation features such as automatic power management by the hardware based on signal strength. By very carefully controlling the transmit power management features, it may be possible to obtain a mean absolute positioning accuracy 1.2 m [8]. However, a similar accuracy may also be achieved using simpler cell-based methods and we therefore do not discuss that method further in this paper.

Multilateration-based localization is based on measuring the time intervals between the transmission of a pulse from the traveler and its and reception at multiple receivers. Such measurements require features not supported in the low-cost commodity devices we wish to use. Further, even if such features are available, the Bluetooth specification permits a clock jitter of 10 microseconds, which translates into a measurement error of roughly three kilometers, making it unsuitable for most localization applications. However, multilateration is an attractive alternative when using slower signals, such as sound.

Cell-based methods determine the location of the traveler based on only the visibility of beacons, without using any distance or angle measurements. Localization is based on the knowledge of the limited range of each of the beacons, allowing the traveler to be localized to the region of intersection of the ranges of all visible beacons. Given the problems with many of the other methods, cell-based methods are quite popular for Bluetooth, as well as for RFID and infrared (IR) technologies.

For example, consider five beacons, A, B, C, D, and E, with ranges of varying shapes and sizes, as suggested by Figure 1. Now suppose the traveler is at some location from which beacons B, C, and D are visible, while beacons A and E are not visible. We may conclude that the traveler is located in the shaded region. Note that we use information on both visibility and non-visibility of beacons to determine the region of the traveler.

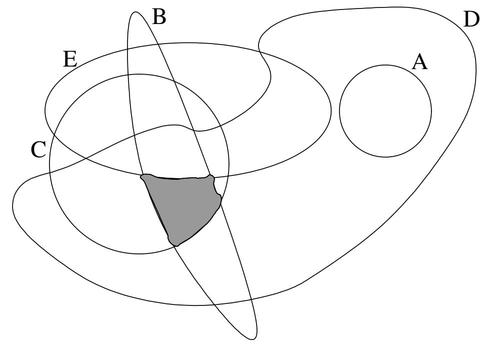


Fig. 1. Cell-based localization: A mobile receiver that is in the ranges of beacons B, C, and D but not in the range of A and E must be located in the shaded region.

An important consideration in cell-based methods is the shape and size of each cell, i.e., the range of each beacon. In outdoor applications, it is often reasonable to assume cells of a regular shape, such as a ellipsoid. However, in indoor applications, such an assumption is not realistic due to channeling and other artifacts of buildings and their contents. For example, it is common for Bluetooth signals to travel along hallways for large distances, but to attenuate rapidly in a transverse direction due to intervening walls and equipment. Similarly, interior windows and doorways, and details such as the construction material, all significantly affect the range of Bluetooth beacons in different directions. The irregular shapes of the cells in the example suggested by Figure 1 are motivated by this observation. For this reason, in this paper we do not assume Bluetooth cells of any particular geometric shape or size. As we will see in the next section, our combinatorial formulation of the problem allows us to model completely arbitrary cell shapes, including ones with holes and other complex features.

If we assume that the location and range of each Bluetooth beacon in an indoor environment is known, then it is simple to determine the location of a traveling mobile device based on the visibilities of beacons using the method illustrated by the example above. Some questions remain, such as how to efficiently store and access the beacon data, and how to provide higher level services, such as mapping and navigation, based on the low-level location information; however, they are not the focus of this paper. Instead, in the next section we turn our attention to the design problem of where to place beacons in order to achieve effective localization.

## III. BEACON PLACEMENT

The cell-based method outlined in the previous section is based on computing the region of intersection of the ranges of the visible beacons and the complements of the ranges of the non-visible beacons. Intuitively, a higher density of beacons is likely to result in smaller intersections, providing more accurate localization. However, the maximum density of beacons for a given application is limited by the total number of beacons that can be deployed, which in turn is limited by constraints on the budget and deployment effort. Therefore, the main question addressed by this section is: Given a limited number of beacons, how should they be placed, in a specified indoor environment, in order to achieve the best results? We make this question more precise below, after covering some preliminaries.

In order to easily model beacon ranges of arbitrary shapes and sizes, we model the problem combinatorially instead of geometrically. In this model, each location that we wish to distinguish is assigned a unique name, or identifier. We refer to these names as *interesting locations* or, for brevity, simply locations. Our work does not make any assumptions on the manner in which interesting locations are determined and named, and these tasks may be completed in a variety of ways. For instance, we may divide an indoor space into a regular grid, with each grid point being an interesting location. Alternately, we may use the floor-plan of a building and assign an interesting location to each room and hallway.

For example, consider the simple floor plan suggested by Figure 2, composed of a large room in the center, a hallway all around it, and smaller rooms along the perimeter. Each of the letters $a$ through $o$ corresponds to an interesting location. We note that some rooms may be deemed to be not interesting for localization, such as the one in the top right corner, perhaps because they are accessible only to very few people. Other rooms, such as the one in the lower right corner, may include multiple locations, perhaps because they are large rooms with multiple zones for different purposes. Similarly, hallways may include multiple locations.

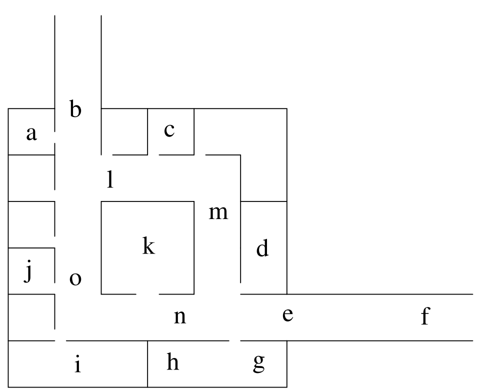


Fig. 2. A simple floor plan, indicating rooms, entrances, and hallways. The letters mark interesting locations for localization purposes.

We model beacon positions in a similar manner, by assigning a name to each potential beacon position. In order to avoid confusion between the points used for localization of the traveler (i.e., the interesting locations) and the points used for beacons, we reserve the use of the term location for the former, and refer to the latter as *candidate beacon positions* or, briefly, beacon positions. We associate a range with each beacon position. This range is the set of locations from which a beacon placed at that position is visible. The range associated with each beacon position depends on several complex factors, such as the geometric relationships between the position and various locations, the presence of obstacles, reflections, and construction materials. However, our combinatorial model allows us to abstract away these complex factors and to focus on their net effect, as determined by empirical observations. All we need is an enumeration of the locations from which each beacon position is visible. For example, we may consider eight candidate beacon positions for the simple floor plan of Figure 2. The corresponding ranges are suggested in Figure 3 using closed curves: The set of locations enclosed within each of the curves in the figure represents the range of a beacon position; the beacon position itself is not depicted.

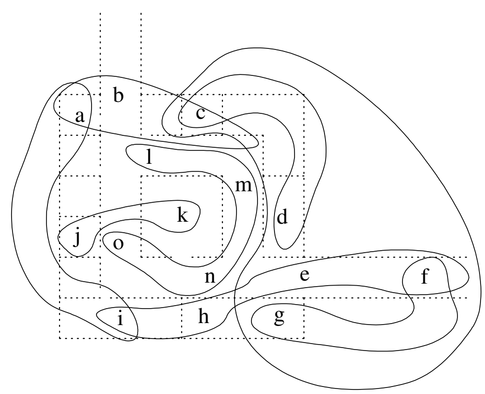


Fig. 3. A representative collection of beacon ranges for the floor plan of Figure 2. The range of each beacon is indicated by a closed curve enclosing the locations (labeled with letters) from which it is visible. The beacon positions are not depicted. The figure is naturally interpreted as a beacon hypergraph.

We model the design space of candidate beacon positions using a hypergraph, called the *beacon hypergraph*, whose vertices represent interesting locations and whose hyperedges represent candidate beacon positions. The hyperedge representing a candidate beacon position $p$ contains the vertices representing the interesting locations from which a beacon placed at $p$ is visible.

Following standard definitions [9], a *hypergraph* $G = (V, E)$ consists of a finite set $V$ of *vertices* and a set $E \subseteq 2^V$ of *hyperedges*, with $\emptyset \notin H$. Intuitively, hypergraphs are generalizations of graphs: In a graph, an edge (excluding loops) is incident on exactly two vertices, while in a hypergraph, a hyperedge may be incident on the vertices of any nonempty set. In pictorial representations of a hypergraph, we use dots or labels for the vertices; each hyperedge is represented by a closed curve that encloses its incident vertices. If we interpreting the closed curves in Figure 3 as hyperedges in this manner then that figure represents a hypergraph $G_1 = (V_1, E_1)$ with vertices $V_1 = \{a, b, \ldots, o\}$ and hyperedges

$$E_1 = \left\{ \begin{array}{llll} \{a,b,c\}, & \{c,d\}, & \{f,g\}, & \{c,d,e,f,g\}, \\ \{e,f,h,i\}, & \{a,i\}, & \{j,k\}, & \{l,m,n,o\} \end{array} \right\}$$

The *complement* $\bar{e}$ of a hyperedge $e$ of a hypergraph $G = (V, E)$ is the hyperedge containing the vertices in $V$ that are not in $e$: $\bar{e} = V \setminus e$. Let $F = \{f_1, f_2, \ldots, f_k\} \subseteq E$ be an arbitrary subset of the hyperedges of $G = (V, E)$ and let $\emptyset \subsetneq X \subseteq V$ be an arbitrary *nonempty* subset of the vertices of $G$. We say that $X$ *is a region induced by* $F$, and $F$ *induces region* $X$, if $X = V \cap g_1 \cap g_2 \cap \cdots \cap g_k$ where each $g_i$ is either $f_i$ or $\bar{f}_i$. It follows that the empty set of hyperedges induces only one region: the set of all vertices in $V$. We use $R(F)$ to denote the set of all *regions induced by* $F$. In the above definition, each set of $k$ choices, fixing either $f_i$ or $\bar{f}_i$ for $g_i$ with $1 \leq i \leq k$, yields at most one region. Therefore, $R(F)$ is composed of at most $2^k$ regions, where $k$ is the number of hyperedges in $F$. In general, $R(F)$ contains fewer than $2^k$ regions because some of the choices for $g_i$ lead to empty sets which, by definition, are not regions.

The regions induced by a set of hyperedges representing beacon ranges correspond exactly to the regions to which a traveler may be localized based on beacon visibility using the method of Section II. Therefore, we may formalize the problem of selecting $n$ most advantageous beacon positions as the problem of selecting a set of $n$ hyperedges of the beacon hypergraph to maximize the number of induced regions:

> *Maximum-Resolution Sub-Hypergraph (MRSH)*: Given a hypergraph $G = (V, E)$ and a non-negative integer $n$, find a subset $F \subseteq E$ such that $|F| \leq n$ and, for all subsets $F' \subseteq E$, if $|R(F')| > |R(F)|$ then $|F'| > n$, where $R(F)$ is the set of regions induced by $F$.

## IV. Maximum-Resolution Sub-Hypergraphs

In this section, we discuss how to solve the maximum-resolution sub-hypergraph problem motivated and formalized above. Perhaps the simplest methods are those based on enumerating the choices for beacon positions. However, if we are to choose $n$ beacon positions from $m$ candidate positions, examining the $\binom{m}{n}$ possibilities is completely impracticable for even modest values of $m$ and $n$. We must therefore seek alternate, more efficient, methods.

*Algorithm G1*: This simple, greedy algorithm selects hyperedges one at a time. At each step, it selects a hyperedge that maximizes the number of induced regions. Any ties are broken arbitrarily, say, be selecting the position with the smallest identifier.

At each step, the hyperedge that maximizes the number of induced regions is one that divides the largest number of currently induced regions. We refer to the number of currently induced regions that are divided by a hyperedge as its *score*.

Returning to the example suggested by Figure 3, suppose we wish to select three hyperedges out of the eight depicted there; i.e., $n = 3$ and $m = 8$. For convenience, let each hyperedge be identified by the string obtained by concatenating its incident vertices in alphabetical order. Thus the eight hyperedges are $abc$, $cd$, $fg$, $cdefg$, $efhi$, $ai$, $jk$, and $lmno$. In the first step, all hyperedges have the same score, 1, since each divides only the single region induced by the empty set of hyperedges. We select the one with the smallest identifier: $abc$. In the second step, hyperedges $cd$, $cdefg$, and $ai$ have score 2 because each divides both the regions induced by $abc$ (the regions inside and outside $abc$). The rest of the hyperedges have score 1. We therefore select $ai$ as the next hyperedge. In a similar manner, $cd$ is the third and final hyperedge selected by Algorithm G1.

In the above example, we may verify, by enumeration, that the solution composed of hyperedges $abc$, $ai$, and $cd$, which induces six regions, is optimal, although, as is typical, it induces fewer than the maximum, $2^3$ regions. However, as illustrated by the following example, the optimality is not guaranteed in the general case. Consider the hypergraph suggested by Figure 4. Each region of that figure is assumed to contain at least one vertex, but the vertices are otherwise unimportant and are omitted from the diagram. Suppose we wish to select five beacon positions from the six candidate positions; i.e., $n = 5$ and $m = 6$. Algorithm G1 selects, in order, hyperedges $b_1$, $b_2$, $b_3$, $b_4$, and $b_5$, yielding a solution that induces 15 regions. However, the alternate solution $\{b_1, b_2, b_4, b_5, b_6\}$ induces 16 regions.

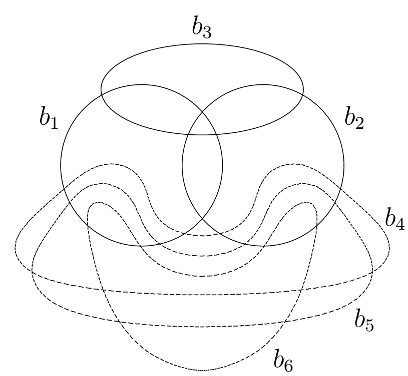


Fig. 4. A hypergraph illustrating the need for non-greedy hyperedge selections. The vertices of the hypergraph are not important and are omitted for simplicity. There is at least one vertex in each of the geometric regions depicted above.

In the example of Figure 4, the difference between number of regions induced by the output of Algorithm G1 and the optimal solution is only one. However, that example is easily generalized into one that results in an arbitrarily large difference. As suggested by Figure 5, we may replace hyperedge $b_5$ in Figure 4 with any number of hyperedges that divide regions in a manner similar to $b_5$, to yield an example with $N$ hyperedges in all. Consider the case of selecting $n = N - 1$ hyperedges from these $N$. It is easy to verify that Algorithm $G_1$ selects the hyperedges $F_1 = \{b_1, b_2, b_3, \ldots, b_{N-1}\}$. We may count the number of regions induced by this set of hyperedges by adding to 1 the score of each hyperedge as it is added to the set: Hyperedges $b_1$, $b_2$, and $b_3$ give a total of 8 regions, as they form a 3-Venn diagram. Hyperedge $b_4$ has a score 3, as it divides the outer region and the regions of $b_1$ and $b_2$. Hyperedge $b_5$ has a score of 4, since it also divides $b_4$. In general, hyperedge $b_k$ has a score of $k-1$, for $4 \leq k \leq N-1$. Thus the total number of regions induced by the output of Algorithm G1 is $R_1 = 8+3+4+\cdots+(N-2) = 5+(N-1)(N-2)/2$. Using similar reasoning, the number of regions induced by the $(N-1)$-set of hyperedges $F_2 = \{b_1, b_2, b_4, b_5, \ldots, b_N\}$ is $R_2 = 4+3+4+5+\cdots+(N-1) = 1+N(N-1)/2 = 1+(N-1)(N-2)/2+(N-1) = R_1+N-5$. Thus the output of Algorithm G1 yields at least $N-5$ fewer regions than the optimal solution.

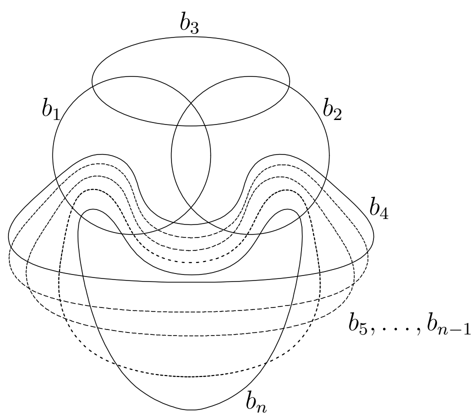


Fig. 5. The example of Figure 4 generalized to $n$ beacons.

Although the above examples illustrate that arbitrarily suboptimal solutions may result from Algorithm G1, in many practical situations, the algorithm may be expected to perform well. We note that an important feature of the example of Figure 5 is that there is a high degree of overlap among hyperedges. More precisely, the maximum degree of a vertex, i.e., the maximum number of hyperedges incident on a vertex, is high. In commonplace scenarios, this number is likely to be small, in turn limiting the worst case sub-optimality of the algorithm. Finally, we note that this sub-optimality does not affect the correctness of the overall method of localization using Bluetooth beacons. Rather, it affects only the granularity of localization, resulting in the possibility that some of the regions to which the traveler is localized may be larger than those possible with an optimal placement of beacons.

## V. Implementation

Recall that one of the important design criteria for our indoor localization system is the use of inexpensive, commodity devices for the beacons, so that they may be easily deployed in large numbers, thereby increasing the accuracy of localization. To that end, each of our beacons consists of essentially a commodity USB Bluetooth adapter attached to a power source. For the power source, we typically use batteries, but line power may be used where convenient. The total cost of each beacon in this setup is under 5 USD: roughly 4 USD for the USB Bluetooth adapter and 1 USD for batteries and connectors.

Unfortunately, this setup is not currently easy to deploy due to a peculiarity of the Bluetooth devices we use: Each time such a device is powered up, putting it into a discoverable mode requires that it be tethered to a USB host such as a desktop computer. Since the devices are untethered in deployment, we need a setup that allows them to be untethered without losing power. To meet this requirement, our current implementation powers the USB Bluetooth device through a USB hub. A representative setup is depicted in Figure 6. Although the addition of the USB hub increases both size and cost of the beacons, the overall cost is still low, at around 10 USD. Further, for a production deployment, the USB Bluetooth devices may be modified so that they do not need the hub.

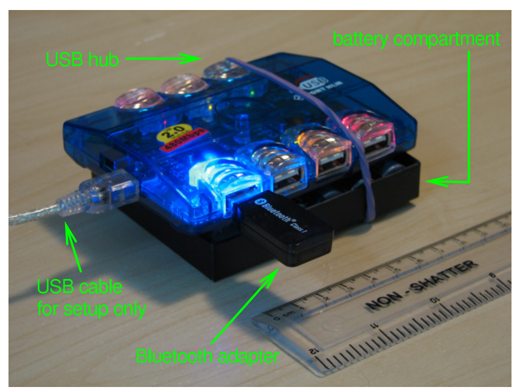


Fig. 6. Prototype Bluetooth beacon.

Another important aspect of our localization method is we perform all the computations required for localization only on the traveler's mobile device, such as a mobile phone or PDA. Apart from deploying the beacons as described above, no other infrastructure support is required. Not only is no computation required at the beacons, no general purpose computation is even possible, since each beacon is simply a powered USB Bluetooth adapter with no host connected to it. Apart from the obvious cost and deployment advantages, this strategy has important ramifications for location privacy [10]. Our implementation requires the traveler's mobile device only to detect the beacons, which advertise themselves in discoverable mode. No Bluetooth connection is made. The Bluetooth hardware in the traveler's mobile device does not need to be in discoverable mode and thus does not need to advertise its presence.

One drawback of Bluetooth-based localization methods is that device discovery is inherently a slow process. The discovery process prescribed by the Bluetooth standard requires a total of 10.24 seconds for completion. We may mitigate this problem by terminating discovery early, especially after a few beacons have been discovered. However, doing so risks a loss of location accuracy, since some beacons may be slow to respond. A promising possibility here is to use an adaptive scheme that mixes long discovery periods with shorter ones in a ratio determined by the number of recently encountered beacons.

## VI. Related Work

Our work on indoor localization has been motivated by recent work on marker-based localization for pedestrians in indoor and outdoor settings, notably M-CubITS [11], [12], [13], [14], [15]. Several authors have studied indoor localization using Bluetooth and related technologies, but have not addressed the placement problem of this paper [2] [8] [4] [16] [3] [5] [6] [7] . In prior work, we have presented a marker-based localization method that uses a short sequence of recently encountered markers, instead of only the currently visible markers [17]. A similar method may be profitably combined with the methods of this paper to yield further improvements in localization.

Problems similar to the maximum-resolution sub-hypergraph problem of Section III have been studied for graphs; however, those results are not directly applicable to our problem because they assume visibility based on graph distance. The problem of placing the fewest possible *landmarks* in a graph so that each node is uniquely identified by its distance from the landmarks, essentially the problem of identifying a metric basis, has been shown to be approximable to a logarithmic factor [18]. Minimum-size *t-Identifying Codes* and *t-Locating-Dominating Codes* are known to be approximable to a logarithmic factor, with sub-logarithmic approximation ratios being intractable [19]. The related problem of identifying codes based on neighborhoods is also known to be NP-complete and approximable to a logarithmic factor [20].

## VII. Conclusion

We motivated the problem of indoor localization in general, and indoor localization using Bluetooth beacons in particular. Our localization scheme is based on two important features of Bluetooth devices: First, they are very inexpensive; therefore, it is practicable to deploy them in large numbers in an indoor environment. Second, they have a range of only a few meters; therefore, the simple visibility of a beacon can be used effectively for localization. We then addressed the problem of beacon placement. We formalized this problem combinatorially as the problem of finding a maximum-resolution sub-hypergraph. An important feature of our formalization is that it does not assume beacon ranges have a simple geometric shape; the shape and size of the range of each beacon is completely arbitrary and may be specified based on the observed characteristics. We presented a simple solution to the problem based on a greedy approach. In continuing work, we are conducting field tests and studying alternate beacon placement algorithms.

## References


[1] B. Hofmann-Wellenhoff, H. Lichtenegger, and J. Collins, *GPS: Theory and Practice*. New York: Springer-Verlag, 1994.

[2] Timothy M. Bielawa, "Position location of remote bluetooth devices," Master's thesis, Virginia Polytechnic Institute and State University, June 2005.

[3] Josef Hallberg, Marcus Nilsson, and Kare Synnes, "Positioning with Bluetooth," in *Proceedings of the 10th International Conference on Telecommunications*, 23 Feb–1 March 2003, pp. 954 – 958.

[4] Kiran Thapa and Steven Case, "An indoor positioning service for Bluetooth ad hoc networks."

[5] G. Anastasi, R. Bandelloni, M. Conti, F. Delmastro, E. Gregori, and G. Mainetto, "Experimenting an indoor Bluetooth-based positioning service," *Proceedings of the 23rd International Conference on Distributed Computing Systems Workshops*, pp. 480 – 483, 19-22 May 2003.

[6] Rune Bentsen, Thomas Alfred Soby Dalsgaard, and Anderson Bo Pedersen, "findIT: Bluetooth positioning project."

[7] Silke Feldmann, Kyandoghere Kyamakya, Ana Zapater, and Zighuo Lue, "An indoor Bluetooth-based positioning system: Concept, implementation and experimental evaluation," in *Proceedings of the International Conference on Wireless Networks (ICWN)*, Las Vegas, Nevada, June 2003, pp. 109–113.

[8] Sheng Zhou and John K. Pollard, "Position measurement using Bluetooth," *IEEE Transactions on Consumer Electronics*, vol. 52, no. 2, p. 555, May 2006.

[9] Claude Berge, *Graphs and Hypergraphs*. Amsterdam, Netherlands: North-Holland, 1973.

[10] Sudarshan S. Chawathe, "Control of personal location data," in *Proceedings of the Location Privacy Workshop*, Schoodic Peninsula, Acadia National Park, Maine, Aug. 2004.

[11] Tetsuya Manabe, Seiji Yamashita, and Takaaki Hasegawa, "On the M-CubITS pedestrian navigation system," in *Proceedings of the 9th International IEEE Conference on Intelligent Transportation Systems (ITSC)*, Toronto, Canada, Sept. 2006, pp. 793–798.

[12] Je-Yeon Kim and Takaaki Hasegawa, "An experimental study on the positioning by M-CubITS," in *Proceedings of the 7th International IEEE Conference on Intelligent Transportation Systems (ITSC)*, Washington, D.C., Oct. 2004, pp. 977–981.

[13] Seiji Yamashita and Takaaki Hasegawa, "On the M-CubITS pedestrian navigation system by a camera-equipped mobile phone," in *Proceedings of the 7th International IEEE Conference on Intelligent Transportation Systems (ITSC)*, Washington, D.C., Oct. 2004, pp. 714–717.

[14] ——, "On the M-CubITS pedestrian navigation system using textured paving blocks and its experiments," in *Proceedings of the 8th International IEEE Conference on Intelligent Transportation Systems (ITSC)*, Vienna, Austria, Sept. 2005, pp. 121–125.

[15] Je-Yeon Kim and Takaaki Hasegawa, "On repositioning of the M-sequence lane markers system in highways and applications in intersections," in *Proceedings of the 6th International IEEE Conference on Intelligent Transportation Systems (ITSC)*, Shanghai, China, Oct. 2003, pp. 520–523.

[16] Kenneth C. Cheung, Stephen S. Intille, and Kent Larson, "An inexpensive Bluetooth-based indoor positioning hack."

[17] Sudarshan S. Chawathe, "Marker-based localizing for indoor navigation," in *Proceedings of the 10th International IEEE Conference on Intelligent Transportation Systems (ITSC)*, Seattle, Washington, Oct. 2007.

[18] Samir Khuller, Balaji Raghavachari, and Azriel Rosenfeld, "Landmarks in graphs," *Discrete Applied Mathematics*, vol. 70, pp. 217–229, 1996.

[19] Jukka Suomela, "Approximability of identifying codes and locating-dominating codes," *Information Processing Letters*, vol. 103, pp. 28–33, Feb. 2007.

[20] Moshe Laifenfeld, Ari Trachtenberg, and Tanya Y. Berger-Wolf, "Identifying codes and the set cover problem," in *Proceedings of the 44th Annual Allerton Conference on Communication, Control, and Computing*, Urbana-Champaign, Illinois, Sept. 2006.